

Equivariant Immersion of Flag Manifolds

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Introduction. In this paper we give applications of Theorem 0.5 of [GHM] (see Section 3 of this paper). The spaces we consider are real flag manifolds $G_{n_1, \dots, n_k} = 0/H$, where $0 = 0(n_1 + \dots + n_k)$ and $H = 0(n_1) \times \dots \times 0(n_k)$. There are many canonical embeddings of these manifolds in one another: G_{n_1, \dots, n_k} embeds in G_{p_1, \dots, p_ℓ} if there is a partition $\{1, \dots, k\} = S_1 \cup \dots \cup S_\ell$ with

$$p_i \geq \sum_{j \in S_i} n_j.$$

In Section 1 we show that any cyclic group of odd order acts freely on G_{n_1, \dots, n_k} if at least two n_i 's are odd.

Our main results follow:

Theorem 0.1. *If*

- (i) G_{n_1, \dots, n_k} *immerses smoothly in* G_{p_1, \dots, p_ℓ}
- (ii) *all* p_i *are odd,*
- (iii) *the codimension* $q = \sum_{i < j} p_i p_j - \sum_{s < t} n_s n_t$ *is odd, and*
- (iv) $q \geq \left[(1/2) \sum_{s < t} n_s n_t \right] + 1,$

then there exists an immersion $G_{n_1, \dots, n_k}/G \subseteq G_{p_1, \dots, p_\ell}/G$ *for any smooth free action of a group* G *of odd order.*

Corollary 0.2. *If* $n_1, \dots, n_k, p_1, \dots, p_\ell$ *are as above then there exists an equivariant immersion* $G_{n_1, \dots, n_k} \subseteq G_{p_1, \dots, p_\ell}$ *for any smooth free actions of a group* G *of odd order.*

Remark 1. It requires some effort to find numbers giving the best possible results in Theorem 0.1 and Corollary 0.2. Such an example is $G_{1,1,1,3} \subseteq G_{1,1,9}$, $G_{1,2,5} \subseteq G_{3,9}$ is close.

Remark 2. Our results can be generalized to complex and quaternionic flag manifolds.