

Interpolation of Operators of Mixed Weak-Strong Type between Rearrangement Invariant Spaces

MARIO MILMAN

1. Introduction. The problem of obtaining sharp interpolation theorems for operators that satisfy weak and strong type conditions has remained open in the general context of rearrangement invariant spaces. Interpolation theorems for operators that satisfy weak type conditions on r.i. spaces were obtained in [16] using a generalization of the methods of Calderón [4], and the mixed weak-strong type problem was treated in [7] in the context of Orlicz spaces and in [18], [19] in the context of $\Lambda(C, \phi)$ spaces.

The purpose of this paper is to compute the K functional for pairs of r.i. spaces (X, Y) whenever X is a Lorentz $\Lambda(X)$ space or a Marcinkiewicz $M(X)$ space. These results generalize to the context of r.i. spaces the work of Holmstedt [6], Jodeit-Torchinsky [7] and Torchinsky [18]. A special case was treated by the author in [12].

Our results have interesting applications. In §6 we obtain a characterization of operators that satisfy mixed weak-strong type conditions on r.i. spaces. As another example we mention that they can be used to prove a general Hausdorff-Young theorem for r.i. spaces without the restrictions imposed in [1] (cf. [12]).

2. Preliminaries. We consider Banach function spaces X of Lebesgue measurable functions on $(0, \infty)$ that are rearrangement invariant (cf. [10], [11]). The fundamental function of a r.i. space is defined by $\phi_X(t) = \|\chi_{(0,t)}\|_X$ where $\chi_{(0,t)}$ is the characteristic function of the interval $(0, t)$. It is shown in [20], [16] that X can be equivalently renormed in such a way that $\phi_X(t)$ is concave, and we shall assume in the following that X has been renormed in this fashion. Moreover, $\phi_X(t)$ is absolutely continuous away from the origin and $\frac{d}{dt}(\phi_X(t)) \leq t^{-1}\phi_X(t)$ a.e. We also assume that there exists a constant $\theta > 0$ such that

$$\int_0^t \phi_X(s) \frac{ds}{s} \leq \theta \phi_X(t), \quad \forall t > 0.$$