

Positive Solutions of Semilinear Schrödinger Equations in Exterior Domains

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1. Introduction. Positive solutions of semilinear Schrödinger equations (see (1) below) will be constructed in an exterior domain $\Omega \subset R^n$, $n \geq 2$, under suitable growth and regularity hypotheses. The specific criteria derived in §5 for the existence of positive solutions are combined with the authors' earlier results [8] to give necessary and sufficient conditions for superlinear Schrödinger equations to be oscillatory in exterior domains. These results apply in particular to the generalized Emden-Fowler equation (21). They constitute the only known characterizations of oscillatory partial differential equations, and can be regarded as extensions of Atkinson's theorem [3] to n -dimensions.

A theorem given by Amann [2] in 1971 provides conditions for the existence of a solution u of (1) in a bounded domain which is squeezed between a subsolution and a supersolution; this is formulated as Theorem A below when specialized to our structure. Amann's theorem is used in §3 to construct a non-increasing sequence of positive solutions u_j in bounded subdomains $G(a, a + j)$ of Ω , $j = 1, 2, \dots$. We then prove (in §4) that this sequence converges to a positive solution of (1) in Ω by employing interior Schauder estimates for solutions of boundary value problems [5] and L^p estimates of Agmon, Douglis, and Nirenberg [1]. We also give a variant of this result in which the existence of a *uniformly* positive solution of (1) in Ω is established. Specific criteria for nonoscillation of (1) can then be derived (§5) on the basis of Atkinson's one-dimensional result [3]; their sharpness is indicated by the fact that they are characterizations in certain cases.

2. Preliminaries. Points in n -dimensional Euclidean space R^n are denoted by $x = (x_1, \dots, x_n)$, and the Euclidean norm of x by $|x|$. The following notation will be used throughout without further comment:

$$S_a = \{x \in R^n : |x| = a\}, \quad a > 0$$

$$G(a, b) = \{x \in R^n : a < |x| < b\}$$

$$G_a = G(a, \infty).$$