

C-Algebras Generated by Weighted Shifts*

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Introduction. This paper is concerned with studying C^* -algebras generated by weighted shifts. In the first section we prove that any weighted shift of finite multiplicity which is bounded below always generates a strongly amenable C^* -algebra. In particular if the shift has scalar weights then the C^* -algebra contains the compact operators. The second section is devoted to finding conditions under which the C^* -algebra is simple modulo compacts and the quotient possesses a unique trace. A systematic treatment of related topics is found in a sequence of papers by J. W. Bunce and J. A. Deddens and in a paper by D. P. O'Donovan cited in the references.

1. Notation. H_0 denotes a separable Hilbert space and $\{A_n, n \geq 0\}$ denotes a sequence of positive invertible operators on H_0 such that

$$\sup \{\|A_n\|, \|A_n^{-1}\|, n \geq 0\} < \infty.$$

If $H = H_0 \oplus H_0 \oplus \cdots$ then P_n denotes the orthogonal projection of H on the n -th copy of H_0 and T denotes the weighted shift defined by $T\{x_0, x_1, \cdots\} = \{0, A_0x_0, A_1x_1, \cdots\}$ where $x_i \in H_0$ and

$$\sum_{i=0}^{\infty} \|x_i\|^2 < \infty.$$

$C^*(T)$ denotes the C^* -algebra generated by T and the identity operator I . If \mathcal{A} is a C^* -algebra, G is a locally compact abelian group and α is a homomorphism of G into $\text{Aut}(\mathcal{A})$ then $G \times_{\alpha} \mathcal{A}$ or $C^*(\mathcal{A}, G)$ denotes the crossed product of G with \mathcal{A} or the covariance algebra corresponding to (\mathcal{A}, G, α) . For more detailed notation and terminology relevant to this section, see [5, page 12].

The main result of this section is the following.

Proposition 1.1. *Let T be a unilateral weighted shift which is bounded below. Moreover suppose that either one of the following conditions holds:*

- (i) *The weights $\{A_n, n \geq 0\}$ act on a finite-dimensional space.*
- (i)' *The weights $\{A_n, n \geq 0\}$ form a commuting family of operators on a separable space.*

Then $C^(T)$ is strongly amenable.*