$\mathfrak{p}$-adic Linear Groups with Ergodic Affine Transformations

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Introduction. It has been conjectured that only compact groups can possess automorphisms which are ergodic with respect to Haar measure. This question was first raised by P. Halmos [8, page 29] and was later investigated in such papers as [12], [15], [17], [20], [24] and most recently in [2]. In this paper we are concerned with an analogous question for a wider class of mappings on a locally compact group, namely affine transformations. These mappings were introduced into ergodic theory on compact abelian groups by F. J. Hahn [6] and were studied in that context in [9] and [22] and on arbitrary compact groups in [16] and [20]. For noncompact, nondiscrete, locally compact abelian groups, it was shown independently in [17], [18] and [20] that no affine transformation can be ergodic. That the same result is valid for connected nonabelian groups was proved by R. K. Thomas [20]. On the other hand, a detailed study of the question of ergodicity of affine transformations on discrete groups was carried out by D. Jonah and the author [11]. In particular, it was shown in [11] that the only infinite discrete groups possessing ergodic affine transformations are the cyclic group Z and the dihedral group D_∞.

It follows from the result of Thomas on connected groups cited above that our problem centers around the study of affine transformations on totally disconnected groups (see Theorem 1 below). The main result of the present work (Theorem 3) is that if G is a linear algebraic group defined over a $\mathfrak{p}$-adic field k of characteristic zero and H is a locally compact group possessing a continuous homomorphism ρ onto a sufficiently large subgroup of the group $G(k)$ of k-rational points of G such that $\ker \rho$ is a solvable extension of a compact group, and if H possesses an ergodic affine transformation, then H is compact.

Ergodic affine transformations. Let G be a locally compact group, λ be a Haar measure on G, and T be a bicontinuous affine transformation on G, *i.e.*, $T(x) = a\tau(x)$, $x \in G$, for some $a \in G$ and $\tau \in \mathrm{Aut}(G)$. The mapping T is called *ergodic* if $\lambda(A)\lambda(A^c) = 0$ for any measurable subset A of G such that $T(A) = A$. More generally, if X is any Hausdorff topological

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