

Local Existence and Nonexistence for Semilinear Parabolic Equations in L^p

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1. Introduction. In this paper we study the initial value problem

$$(1.1) \quad u'(t) = Au(t) + J(u(t)), \quad u(0) = \phi,$$

where u is a curve in a Banach space E , $u: [0, T] \rightarrow E$, A is the infinitesimal generator of a C_0 semigroup e^{tA} on E , and J is a nonlinear function from a subset of E into E . We are primarily concerned with the case where $E = L^p(\Omega)$, Ω is a domain in R^n and $1 \leq p < \infty$, A is an elliptic operator on Ω , and J has polynomial growth, e.g. $Ju = |u|^{p-1}u$ for some $p > 1$.

The semilinear equation (1.1) has certainly been the object of considerable study. In recent years Ball [1], [2], Henry [4], Kielhöfer [9], [10], Pecher [11], and Webb [17], among others, have dealt with (1.1) in the parabolic case. Our treatment differs from these works in that we wish to allow the initial value $u(0)$ to be an arbitrary function ϕ in $L^p(\Omega)$. Of course, we can only require that the solution to (1.1) be differentiable in $L^p(\Omega)$ for $t > 0$. However, it will be continuous in $L^p(\Omega)$ for $t \geq 0$.

As is standard practice, we study (1.1) via the corresponding integral equation

$$(1.2) \quad u(t) = e^{tA} \phi + \int_0^t e^{(t-s)A} J(u(s)) ds.$$

In proving the existence of solutions to (1.1), we first prove the existence of continuous solutions to (1.2) and then show that these solutions are differentiable for $t > 0$ and satisfy (1.1).

In some respects this paper is a sequel to [18], a previous work by the author. Results and techniques from [18] are sometimes referenced; but apart from these occasional references, one can read the present paper without having previously read [18].

The outline of this paper is as follows: In Section 2 we prove existence of solutions to (1.1) and (1.2) in an abstract setting. In Section 3 we apply these results to a large class of examples, including polynomial perturbations to elliptic operators which are generators of analytic semigroups on $L^p(\Omega)$. Initial values are in some cases allowed to be arbitrary functions in $L^p(\Omega)$. The basic idea in these two sections is to balance smoothing properties of