

# *Some Two Parameter Problems for Singularly Perturbed Equations of Elliptic Type*

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**1. Introduction.** We consider here some results on the existence and the asymptotic behavior of solutions of the singularly perturbed nonlinear problems

$$(1.1) \quad \varepsilon \Delta u = \mu A(x, u) \cdot \nabla u + h(x, u)$$

and

$$(1.2) \quad \varepsilon \Delta u = \mu \sum_{j=1}^N b_j(x, u) u_{x_j}^2 + h(x, u)$$

for  $x$  in a bounded region  $\Omega$  in  $\mathbf{R}^N$  ( $N \geq 2$ ) which satisfy along the boundary of  $\Omega$  either Dirichlet—or Robin—type conditions. The parameters  $\varepsilon$  and  $\mu$  are assumed throughout to be small and positive and to tend to zero in an interrelated fashion. We will determine precisely how the relative sizes of  $\varepsilon$  and  $\mu$  influence the behavior of solutions of (1.1) and (1.2), especially near the boundary of  $\Omega$ . At the same time we will give estimates on these solutions in regions where up to now complete information has been lacking. Such regions often have corners and/or boundaries which coincide with solutions of certain lower-order equations (that is, characteristic boundaries).

Problems of this type were first studied for linear differential equations in two dimensions (that is,  $A(x, u) = (a(x, y), b(x, y))$  and  $h(x, u) = h(x, y)u + g(x, y)$  in (1.1)) by O'Malley in a series of papers [16–18]. More recently, Butuzov [3, 4] has considered this linear equation with Dirichlet boundary data in a rectangle in order to examine the interaction of the boundary with the characteristic curves of the “semi-reduced” equation  $\mu(a(x, y), b(x, y)) \cdot (u_x, u_y) + h(x, y)u + g(x, y) = 0$ . Finally van Harten in his dissertation [20] and in [21] has discussed certain linear and nonlinear equations of the form (1.1) in dimensions greater than two when the parameter  $\mu$  is a power of  $\varepsilon$ .

The article of O'Malley [18], the monograph of Lions [15], and the dissertation of van Harten [20] also contain interesting discussions of related singular