

Spectral Properties of Real Parts of Weighted Shift Operators

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1. Introduction. Let \mathcal{H} be an infinite dimensional separable Hilbert space with fixed orthonormal basis $\{\phi_n\}_{n=1}^\infty$. Suppose that $C = \int \lambda dE_\lambda$ is a bounded self-adjoint operator with the following matrix representation with respect to the given basis:

$$(1.1) \quad C = \frac{1}{2} \begin{bmatrix} 0 & a_1 & 0 & 0 & 0 & \dots \\ a_1 & 0 & a_2 & 0 & 0 & \dots \\ 0 & a_2 & 0 & a_3 & 0 & \dots \\ 0 & 0 & a_3 & 0 & a_4 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \end{bmatrix}.$$

The operator C is a special case of the Jacobi matrix discussed by Stone [5]. It is the aim of this paper to use the structural properties of C as a Jacobi matrix to analyze its spectral properties.

Since real parts of unitarily equivalent operators are unitarily equivalent, it will always be assumed that the weight sequence $\{a_n\}$ in (1.1) is non-negative. Note that if $a_n > 0$ for all n , then ϕ_1 is a cyclic vector.

2. The eigenvalue problem. This section develops a technique for analyzing the point spectrum of C under the assumption the weight sequence is strictly positive. Appropriate examples will follow. The first three lemmas essentially appear in Stone [5].

Lemma 1. *Each basis element ϕ_n can be expressed in the form $P_n(C)\phi_1$, where $P_n(C)$ denotes a polynomial in C .*

Proof. Clearly $P_1(C) \equiv I$ and $C\phi_1 = (a_1/2)\phi_2$ so that $P_2(C) = (2/a_1)C$. For $k \geq 2$, $C\phi_k = (1/2)[a_{k-1}\phi_{k-1} + a_k\phi_{k+1}]$. Hence $\phi_{k+1} =$