

Involutions Fixing the Disjoint Union of Two Projective Spaces

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This note answers a particular case of the following question posed by Steenrod to P. E. Conner in 1962.

Question. *Given a smooth closed manifold F , not necessarily connected, does there exist a non-trivial differentiable involution T on a smooth closed manifold M with F as its fixed point set? Given a particular F , can one identify all involutions and manifolds (T, M) with F as the fixed point set?*

Of course, in order to have any sort of success, one translates this into a bordism question working with the bordism classes of these manifolds and attempting to identify these actions up to bordism. In this note I will be concerned with a special case of the second question. I want to consider the case $F = RP(n) \cup RP(m)$, the disjoint union. There is a smooth involution on a smooth closed manifold with this F as its fixed point set. Let τ_n be the involution on $RP(n + m + 1)$ defined in homogeneous coordinates by

$$\tau_n [x_0, x_1, \dots, x_{n+m+1}] = [-x_0, \dots, -x_n, x_{n+1}, \dots, x_{n+m+1}].$$

Then $F_{\tau_n}(RP(n + m + 1)) = RP(n) \cup RP(m)$. In general, this is not the only involution fixing F . If $n + m + 1$ is odd, then $RP(n + m + 1)$ bounds and we have that $\Gamma(\{\tau_n, RP(n + m + 1)\})$ has $RP(n) \cup RP(m)$ as its fixed set.

The results of this note show that [3;27.6] fits into a rather general family of smooth involutions. This theorem states that if $F = * \cup S^k$ ($* = \text{point}$), then $k = 1, 2, 4$, or 8 , $r = 2k$, and M^r is bordant mod 2 to the appropriate projective plane.

If we take $F = RP(n)$ the answer to the second question has been known for some time; cf. [3;27.6] and [5]. Also, Capobianco [1] has given the most recent results on the first question.

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