

Biholomorphic Equivalence in a Class of Graph Domains

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0. Introduction. Some of the fundamental problems in several complex variables are questions of biholomorphic equivalence and automorphisms of domains in \mathbb{C}^n . Behnke and Thullen [1] give a comprehensive description of biholomorphic equivalences for Reinhardt and circular domains in \mathbb{C}^2 ; however, they ask the question: Do there exist domains of holomorphy whose automorphism groups contain only the identity map? Burns, Shnider, and Wells [4] construct deformations of the unit ball into such domains, but these are nonspecific or depend on convergent power series expressions for their definitions. The main result of this paper is used to construct specific deformations of the unit ball into algebraically defined strongly pseudoconvex domains without automorphisms.

Let $p: \mathbb{C}^n \rightarrow \mathbb{C}$ be a holomorphic function and consider the open set $D_p = \{z = (z_1, \dots, z_n) \in \mathbb{C}^n: |z_1|^2 + \dots + |z_n|^2 + |p(z)|^2 < 1\}$. We call a connected D_p a graph domain, since it can be obtained by the coordinate projection to \mathbb{C}^n of the intersection of the graph of p and the unit ball $B^{n+1} \subset \mathbb{C}^{n+1}$. Smoothly bounded graph domains are strongly pseudoconvex, and the interplay between the extension theorem of Fefferman [6] and theorems of the Bochner-Hartogs type for such domains reduces biholomorphic equivalence problems to CR-equivalence problems on the boundary. Using a recent result of S. Webster [13] on CR-equivalences, we determine completely the biholomorphic equivalences, including automorphisms, for a class of graph domains defined from polynomials. Within this class, questions of equivalence are answered by unitary linear algebra.

The first section of this paper standardizes notation and reviews some well-known theorems on CR-equivalences. In the second section we use Webster's Theorem to prove that certain varieties in $\mathbb{P}^{n+1}(\mathbb{C})$ are biholomorphically equivalent only by unitary transformations, and from this we prove the theorem on graph domains. In the third section we construct examples of a one-parameter deformation of B^n into biholomorphically inequivalent Reinhardt domains and a one-parameter deformation of B^4 into biholomorphically inequivalent domains without automorphisms.

S. Webster [14] has obtained similar results for a different class of polynomially defined domains using more algebraic techniques.