

More on Weak Estimates of Oscillating Kernels, II

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§0. Introduction. The purpose of this paper is two-fold. The primary problem is that we continue [15] with the discussion (for $t \in \mathbf{R}$) of

$$(0) K_p(t) = |t|^{-1/p} \phi(t), \quad \phi(t) = e^{i\eta(\log|t|)^\eta} (\log(2 + |t|))^{-\delta}$$

$0 < \eta < 1$, $\delta = (1/2)(1 - \eta)$ and $1 < p \leq 2/(1 + \eta)$. In [15] we proved that the point (p, p) is a weak restricted point for K_p but not a strong restricted point. Here, we prove (see Theorem 2 in §3) that the point (p, p) is in fact a weak point. In [18], Zafran was the first to give an example that maps the sequence space l^p into l^p weakly but not strongly. This example is similar to his. Let me add that the techniques I use here are flexible enough that they can be applied toward solving the (L^p, L^q) mapping problem for many of the kernels $K(t) = b(t)e^{ia(t)}$ where $a(t)$, $b(t)$ are real-valued functions and $a(t) \rightarrow \infty$ at infinity. Also, see [5], [10], [11], [12], [14], and [15], I should like to add that the above general problem is a joint project with W. B. Jurkat and is presently being undertaken.

With regard to the general problem stated above, in [6] and along with Stein in [8], C. Fefferman gave a complete solution to the mapping problem for the kernels

$$K(t) = \frac{e^{i|t|^a}}{|t|^b} \operatorname{sgn} t \quad (t \in \mathbf{R}^1), \quad a < 0, \quad 0 < b \leq 1 - \frac{a}{2},$$

(in the cases where $a < 0$, we assume further that K has compact support).

C. Fefferman also knew how to do the cases where $0 < a < 1$ (of course all of his results apply as well to higher dimensions). Besides the kernels that appear here in this paper, Jurkat and I solved the mapping problem for all the cases where $0 < a$, $b \leq 1$. We are at present having success with some of the kernels $K(t) = b(t)e^{ia(t)}$ where $b(t)$ is either increasing or decreasing and $|a''(t)|$ exists and is positive at infinity. In [2] and [13], these authors are looking at similar kernels in higher dimensions. If fact, in [2] and [6] the authors solve the mapping problem for the kernels

$$K(t) = \frac{e^{i|t|}}{|t|^b} \quad (t \in \mathbf{R}^2), \quad \frac{3}{2} < b < 2$$