

Regularity of the Free Boundary of a Gas Flow in an n -dimensional Porous Medium

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1. Introduction. The density $u(x, t)$ of gas in a porous medium satisfies the equation

$$(1.1) \quad \frac{\partial u}{\partial t} = \Delta u^m$$

for $x \in R^n$, $t > 0$, where m is a given number, $m > 1$, and an initial condition

$$(1.2) \quad u(x, 0) = u_0(x)$$

for $x \in R^n$. Here $u_0(x) \geq 0$ and $u(x, t) \geq 0$.

Since the equation (1.1) is a degenerate nonlinear parabolic equation, the concept of a regular solution is too restrictive. By a solution of (1.1), (1.2) we shall mean a function $u(x, t)$ such that, for any $T < \infty$,

$$(1.3) \quad \int_0^T \int_{R^n} [(u(x, t))^2 + |\nabla_x u^m(x, t)|^2] dx dt < \infty$$

and

$$(1.4) \quad \int_0^T \int_{R^n} \left(u \frac{\partial f}{\partial t} - \nabla_x u^m \cdot \nabla_x f \right) dx dt + \int_{R^n} u_0(x) f(x) dx = 0$$

for any continuously differentiable function f with compact support in $R^n \times [0, T)$.

We shall assume that

$$(1.5) \quad 0 \leq u_0(x) \leq N \quad (N < \infty),$$

$$(1.6) \quad \int_{R^n} (u_0(x))^2 dx < \infty.$$

Under these conditions [9] there exists a unique solution of (1.1), (1.2) (in the sense of (1.3), (1.4)). Further, if $u_\eta(x, t)$ is the solution with

$$(1.7) \quad u_\eta(x, 0) = u_0(x) + \eta, \quad \eta > 0,$$