

Approximation Theorems for Stochastic Operators

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This paper deals with the convex set \mathcal{S} of all stochastic operators on $L^1[0,1]$ (see Section 1 for definitions). In Sections 3 and 4 it is shown that the group \mathcal{G} of positive invertible isometries and its convex hull $\text{conv } \mathcal{G}$ are dense in \mathcal{S} for the weak and strong operator topologies, respectively (Theorem 1 and Corollary 1). The strong operator closure of \mathcal{G} is identified as the set of all positive isometries (Theorem 2). This extends corresponding results of J. R. Brown for doubly stochastic operators [1] and is related to C. W. Kim [11] and [12].

In Section 5 we prove that the set \mathcal{O} of all stochastic operators without invariant sets and its subset \mathcal{O}_μ consisting of those possessing an equivalent invariant measure μ are, respectively, norm dense in \mathcal{S} and strong operator residual in the set \mathcal{S}_μ of all μ -preserving stochastic operators (Corollary 3 and Proposition 4). This last result is an analog of the classical result of Halmos on the residuality of ergodic transformations in the measure preserving invertible ones [6]. Next it is shown that “most” stochastic operators are conservative and ergodic (Theorem 3); this can be viewed as an extension of a similar result obtained recently by Choksi and Kakutani for invertible positive isometries [3, Theorem 3].

Section 6 is devoted to operators on $L^2[0,1]$. In particular, it is shown that the positive invertible L^2 -isometries (unlike positive L^1 -isometries) span a strong operator dense linear subspace of all continuous linear operators (Corollary 5).

1. Terminology and notation. Let (X, Σ, m) denote the unit interval with Borel σ -algebra and Lebesgue measure. A function $\tau: X \rightarrow X$ which is $\Sigma - \Sigma$ measurable and nonsingular (i.e. $m(A) = 0 \Rightarrow m\tau^{-1}(A) = 0$) is called a *transformation*. A transformation τ is called *invertible* if τ^{-1} exists and is also a transformation.

Every transformation τ induces an operator in $\mathcal{L}(L^\infty(m))$ defined by $h \rightarrow h \circ \tau$ ($h \in L^\infty(m)$). It is a positive contraction and is easily seen to be the adjoint of a $T_\tau \in \mathcal{L}(L^1(m))$. In fact, if the elements of $L^1(m)$ are