

Generic Properties of Navier-Stokes Equations: Genericity with Respect to the Boundary Values

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Introduction.

We consider the stationary, non-homogeneous Navier-Stokes equation in a bounded open set Ω of \mathbf{R}^n , $n = 2, 3$ with boundary Γ :

For f and ϕ given, to find $u = (u_1, \dots, u_n)$ and p such that:

$$(0.1) \quad \begin{cases} -\nu \Delta u + \sum_{i=1}^n u_i D_i u + \text{grad } p = f & \text{in } \Omega \\ \text{div } u = 0 & \text{in } \Omega \\ u = \phi & \text{on } \Gamma. \end{cases}$$

Here u and p represent the velocity and the pressure of a viscous incompressible fluid filling Ω , the fluid being submitted to stationary volumic forces f , and Γ moving with the time-independent velocity ϕ .

It is well known (cf. Leray [6]) that (0.1) is equivalent to a functional equation involving only u ; we shall denote by $S(f, \nu, \phi)$ the set of solutions (in a function space which will be specified later) of this functional equation.

In [2], [3], C. Foias and the second author have obtained several generic results concerning $S(f, \nu, \phi)$; for instance, for fixed ϕ and ν , this set is finite for every f belonging to a dense open set of the function space to which f belongs.

The aim of this paper is to complete these results, in particular for the case where f and ν are fixed, and this is essential to cover many physical situations where f is 0 and the motion of the fluid is produced by the motion of the boundary. While the generic results given in [2], [3] were obtained by application of the infinite dimensional version of Sard's theorem due to S. Smale [14], the results which we present here follow from a transversality theorem which is due in its different versions to F. Quinn [9], Uhlenbeck [20], and Thom. The version of the transversality theorem which we use is very close from that of F. Quinn [9] and was stated in detail in a previous