

Some Simple C^ -algebras Associated with Free Groups*

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§0. Introduction. Let G be a countable discrete group containing a free, non-abelian subgroup F . We shall prove that under certain additional hypotheses (e.g., F is normal in G with trivial centralizer), the reduced C^* -algebra $C_r^*(G)$ is simple with unique trace (definitions appear in §1). The technique used is a refinement of the proof of [1, Theorem VC]. As corollaries we get that $C_r^*(G)$ is simple with unique trace when G is a non-trivial free product [8, Theorem 1.1] or when G has the form of the groups constructed in [11, Section 4.3] to get examples of non-isomorphic factors. Because of the relationship between crossed products of reduced group C^* -algebras with automorphism groups and semi-direct products [2, Lemma 1], we also are able to prove (Corollary 10) that the reduced crossed product of $C_r^*(F)$ and a group H of outer automorphisms of F is simple (see [3] for a discussion).

Our goal is to show that one powerful technique can be used to show that a wide variety of reduced group C^* -algebras are simple with unique trace. We are indebted to colleagues S. Bachmuth, A. Connes, J. Lewin, H. Mochizuki, D. Solitar and J. Zelmanowitz for helpful discussions.

§1. Notation and preliminaries. We shall use the notation developed in Sections II and V of [1] with certain exceptions. Briefly, G is a countable discrete group with identity e , $L(G)$ its complex group algebra and $L^2(G)$ the completion of $L(G)$ in the L^2 -norm. We write elements of $L(G)$ as formal sums $\sum_{x \in G} \alpha_x x$, where only a finite number of the α_x are non-zero. For $b = \sum_{x \in G} \alpha_x x$ in $L(G)$ set $b^* = \sum_{x \in G} \bar{\alpha}_x x^{-1}$. We write $L^2(G) = \left\{ \sum_{x \in G} \alpha_x x : \sum_{x \in G} |\alpha_x|^2 < \infty \right\}$. The left regular representation of $L(G)$ on the Hilbert space $L^2(G)$ is given by $\left(\sum_{x \in G} \alpha_x x \right) \left(\sum_{y \in G} \gamma_y y \right) = \sum_{x \in G} \sum_{y \in G} \alpha_x \gamma_y xy = \sum_{z \in G} \left(\sum_{w \in G} \gamma_{w^{-1}z} \alpha_w \right) z$. Let $C_r^*(G)$ denote the C^* -algebra generated by this representation of $L(G)$ and use $\|\cdot\|$ to denote