# *A Simple Proof of Mehler's Formula for q-Hermite Polynomials*

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The $q$-Hermite polynomials, $H_n(x|q)$, $|q| < 1$, can be defined by their generating function:

$$\prod_{n=0}^{\infty} \frac{1}{(1 - 2xrq^n + r^2q^{2n})} = \sum_{n=0}^{\infty} \frac{H_n(x|q)r^n}{(q)_r}, \tag{1}$$

where $(a)_r = (1 - a)(1 - aq) \ldots (1 - aq^{r-1})$, $(a)_0 = 1$, or by their recurrence relation:

$$2xH_n(x|q) = H_{n+1}(x|q) + (1 - q^n)H_{n-1}(x|q), \tag{2}$$

$$H_{-1}(x|q) = 0, \quad H_0(x|q) = 1,$$

or by their Fourier expansion:

$$H_n(\cos\theta|q) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} \cos(n - 2k)\theta \tag{3}$$

$$= \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} e^{i(n-2k)\theta},$$

where $\begin{bmatrix} n \\ k \end{bmatrix}$ is the $q$-binomial coefficient,

$$\begin{bmatrix} n \\ k \end{bmatrix} = \frac{(q)_n}{(q)_k (q)_{n-k}}.$$

These polynomials have been studied by Rogers [5] [6] [7] [8], Szegö [9], Carlitz [3] [4] and Askey and Ismail [1]. The Rogers-Ramanujan identities first arose in the study of these polynomials (see Rogers [7], Bressoud [2]). The $q$-Hermite polynomials are also of formal interest, for they have been found to satisfy analogs of many of the formulae known for the Hermite polynomials. Among these is a $q$-analog of Mehler's formula:



Indiana University Mathematics Journal ©, Vol. 29, No. 4 (1980)