

Holomorphic Extension of the Scattering Amplitude for Moving Obstacles

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This paper is a sequel to our paper [2]. We are studying the scattering properties of solutions of

$$(1) \quad u_{tt} - \Delta u = 0 \quad \text{in } Q, \quad u = 0 \quad \text{on } \Sigma = \partial Q$$

where Q is a domain in space-time \mathbf{R}^{n+1} , n odd, $n \geq 3$. The obstacle $\mathcal{O}(t)$ at time t is the t -cross section of the complement of Q .

We continue to assume that $\mathcal{O}(t)$ varies smoothly and remains in a bounded set in \mathbf{R}^n . Under the assumptions $(CP)_+$ and $(LD)_+$ (see below) the scattering operator S is well defined. It has a translation representation $S^\#$ and a spectral representation \tilde{S} , both of which are bounded linear operators on $L^2(R \times S^{n-1})$ where S^{n-1} is the unit sphere in \mathbf{R}^n . The kernel $K^\#$ of $S^\# - I$ is called the echo kernel; the kernel \tilde{K} of $\tilde{S} - I$ is the scattering amplitude. Both $K^\#$ and \tilde{K} are tempered distributions on $\mathbf{R} \times S^{n-1} \times \mathbf{R} \times S^{n-1}$.

In [2] we obtained representations for \tilde{K} and $K^\#$ in terms of the reflections of plane waves $\psi(x, t; \sigma, \omega)$ for *real* frequency σ and direction $\omega \in S^{n-1}$, provided these reflections exist. A sufficient condition for their existence is the uniform decay of local energy $(ULD)_+$.

In this paper we only assume $(CP)_+$ and $(LD)_+$. We prove the existence of $\psi(x, t; \sigma, \omega)$ for complex frequencies σ with $\text{Im } \sigma < 0$. Both $K^\#$ and \tilde{K} can be simply expressed in terms of the asymptotic values of these reflections. We can also express $K^\#$ in terms of the asymptotic values of the reflection of a plane pulse.

It is a consequence of [2, Theorem 13b] that when $\sigma' - \sigma = \lambda$ is real, $\tilde{K}(\sigma', \omega'; \sigma, \omega) = \tilde{K}(\sigma + \lambda, \omega'; \sigma, \omega)$ has a holomorphic extension to $\text{Im } \sigma < 0$ with values in a space of tempered distributions. In this paper we show that $\sigma \rightarrow \tilde{K}(\sigma + \lambda, \omega'; \sigma, \omega)$ takes values in $H^{1-\varepsilon}(\mathbf{R}_\lambda; L^2(S^{n-1} \times S^{n-1}))$. As a further improvement over the results of [2], we find that when the motion of the obstacle has period T , we may express \tilde{K} as a series

$$\tilde{K}(\sigma', \omega'; \sigma, \omega) = \sum_{m=-\infty}^{+\infty} A_m(\sigma, \omega', \omega) \delta(\sigma' - \sigma - m\nu)$$