

On the Theory of Algebra-Valued Generalized Polars

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1. Introduction. Let E (respectively V) denote a vector space (respectively an algebra with identity) over an algebraically closed field K of characteristic zero. In this paper we employ circular cones in our study of the generalized polars of the product of algebra-valued abstract homogeneous polynomials (a.h.p.) from E to V and obtain more general formulations of our own results (see [17, Theorems 2.5, 3.1, 4.2, 4.4]) established earlier in relation to a.h.p.'s from E to K . Similar, but highly specialized, studies were undertaken in the past by Bôcher and Walsh and, later, their results were generalized to a.h.p.'s from E to K by Marden and the author (cf. the paper cited above). All these results, however, become natural corollaries to our present formulation.

Let E and V be vector spaces over the same field K of characteristic zero. A mapping $P: E \rightarrow V$ is called [5, pp. 55, 59], [4, pp. 760–763], [14, pp. 52–61] a *vector-valued abstract homogeneous polynomial* (briefly, *vector-valued a.h.p.*) of degree n if for every $x, y \in E$,

$$P(sx + ty) = \sum_{k=0}^n A_k(x, y) s^k t^{n-k} \quad \forall s, t \in K,$$

where the coefficients $A_k(x, y) \in V$ and are independent of s and t for any given x, y in E . For the distinction of cases, the adjective ‘vector-valued’ in the above definition will be replaced by ‘*algebra-valued*’ or it will be completely dropped according as V is (in particular) an algebra or it is the field K . We denote by \mathbf{P}_n^* the class of all vector-valued a.h.p.’s of degree n from E to V (even if V is an algebra) and, by \mathbf{P}_n , the class of all a.h.p.’s of degree n from E to K . The n^{th} -*polar* of P is the mapping (see [5, pp. 55, 59] or [4, pp. 762–763] for its existence and uniqueness) $P(x_1, x_2, \dots, x_n)$ from E^n to V which is a symmetric n -linear form such that $P(x, x, \dots, x) = P(x)$ for every $x \in E$. The k^{th} -*polar* of P is then defined by

$$P(x_1, \dots, x_k, x) = P(x_1, \dots, x_k, x, \dots, x).$$

From now on we shall assume throughout that K is an algebraically closed field of characteristic zero and that V is an algebra with identity over K