

Complete Integrability and Perturbation of a Nonlinear Dirichlet Problem, II

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Conventional ideas on complete integrability for nonlinear Hamiltonian systems to date have proved of very limited value for the study of partial differential equations in dimensions higher than two. Moreover those approaches centering around the well-known theorems of Liouville (see [1, page 317]) have the additional difficulty that the methods of study used yield no information on the properties of a slightly perturbed problem. In Part I of this work we gave a novel (but in our opinion natural) definition for complete integrability of abstract nonlinear mappings A acting between Banach spaces. We then exhibited a well-known nonlinear elliptic partial differential operator \mathfrak{A} defined on an arbitrary bounded domain Ω of any finite dimension N that was completely integrable in this sense. In fact it was shown that the associated nonlinear Dirichlet problem could be solved explicitly in terms of canonical coordinate changes and that the abstract mapping A was globally equivalent to the normal form of an infinite dimensional fold.

In this part of our research we study the effect of a perturbation of the partial differential operator \mathfrak{A} . Unlike the difficulties in treating perturbations associated with the usual approaches to complete integrability mentioned above, our methods yield results. We show (among other things) that even though complete integrability in our sense may be lost, significant facts about the explicit solutions of the associated nonlinear Dirichlet problem may be obtained. In particular we show that apart from a certain exceptional set, the solutions of the perturbed problem can be accurately approximated by these explicit solutions.

This part of our work is divided in three sections. In Section 1 we specify the perturbation problems to be studied, review the notation of Part I and using this notation prove the a priori estimates necessary to study our perturbation results. In Section 2 we state and prove our main perturbation results for the given operator \mathfrak{A} . Finally in Section 3 we abstract the perturbation problem and prove a general stability problem valid for a large class of C' nonlinear Fredholm operator equations of index zero.