

On the Non-occurrence of Secondary Bifurcations in Eigenvalue Problems for Nonlinear Operators in Banach Space

GEORGE H. PIMBLEY, Jr.

1. Introduction. In this paper we consider a general nonlinear eigenvalue problem:

$$(1) \quad F(x) = \lambda x$$

where λ is a real parameter, $x \in B$ where B is a Banach space over the real field R , with norm $\|\cdot\|$, and F is a nonlinear operator mapping B into itself. We suppose that $F(\odot) = \odot$ where \odot is the null element of B , and that F is *continuous* and *compact*.

Because $F(\odot) = \odot$, Eq. (1) has the trivial solution $x = \odot$. As with any more or less standard eigenvalue problem, we ask for those values of λ such that Eq. (1) has nontrivial solutions. In contrast to such problems for linear operators, these values of λ generally cover intervals in R , and the eigenelements x associated with them form nonlinear manifolds or branches parameterized by λ in these intervals.

Such problems are certainly of interest. Examples are the equation for the buckled beam (cf. [4, page 182]):

$$\int_0^1 G(s, t)x(t)dt \sqrt{1 - \left[\int_0^1 G_s(s, t)x(t)dt \right]^2} = \lambda x(s)$$

and the equation for the deflections of rotating rods [10]:

$$\int_0^1 \int_0^1 G_1(s, \xi) G_2(\xi, t) \sin x(t) \sin x(\xi) dt d\xi = \lambda x(s)$$

where $G(s, t)$, $G_1(s, \xi)$, $G_2(\xi, t)$ are appropriate Green's functions.

The eigenvalues of the linearized problem at the origin

$$(2) \quad F'(\odot)h = \lambda h$$

are of importance in the study of problem (1). Here $F'(x)$ is the Fréchet derivative of F , supposed to exist, [8, page 292]. The compactness of F