

# *Projections onto Spaces of Holomorphic Functions*

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**I. Introduction.** In [3] Kolaski showed, for a large class of measures  $\sigma$ , the Hilbert space generated by the analytic polynomials in  $L^2(\sigma)$  has a reproducing kernel  $C_\sigma$ . In this note we show that the integral operator  $T_\sigma$  with kernel  $C_\sigma$  is a closed projection of  $L^p(\sigma)$  onto the space of holomorphic functions in  $L^p(\sigma)$  ( $1 \leq p < \infty$ ), and that  $T_\sigma$  is unbounded when regarded as an operator on  $L^1(\sigma)$ . Concerning the generality of the projections  $T_\sigma$ , we remark that  $T_\sigma$  is the Bergman projection when  $\sigma$  is taken to be Lebesgue measure on an  $N$ -circular subset of  $\mathbf{C}^N$ .

For a given positive integer  $N$ ,  $\mathbf{C}^N$  will denote the vector space of all ordered  $N$ -tuples  $z = (z_1, \dots, z_N)$  of complex numbers, with inner product

$$(1) \quad \langle z, w \rangle = z_1 \bar{w}_1 + \dots + z_N \bar{w}_N$$

and norm

$$(2) \quad |z| = \langle z, z \rangle^{1/2}.$$

Throughout this paper  $\sigma$  shall *ALWAYS* denote a *CLASS K MEASURE*, i.e.,  $\sigma$  is a non-negative, bounded, Borel measure on  $\mathbf{C}^N$  which satisfies (3), (4) and (5), (Examples and comments can be found in [3].):

$$(3) \quad F_\sigma = \text{Support } \sigma \text{ is compact,}$$

$$(4) \quad F_\sigma \not\subset \bigcup_{i=1}^N \{z_i = 0\},$$

$$(5) \quad \sigma \text{ is invariant under the rotations} \\ (z_1, \dots, z_N) \rightarrow (e^{i\theta_1} \cdot z_1, \dots, e^{i\theta_N} \cdot z_N) \quad (0 \leq \theta_i \leq 2\pi, 1 \leq i \leq N).$$

We associate to each measure  $\sigma$  the complete Reinhardt domain

$$(6) \quad \Omega_\sigma = \text{Interior Holomorphic Hull } F_\sigma$$

and the moments

$$(7) \quad \hat{\sigma}(\alpha) = \int z^\alpha \bar{z}^\alpha d\sigma(z), \quad (>0 \text{ by 4})$$

where  $\alpha = (\alpha_1, \dots, \alpha_N) \in \mathbf{Z}_+^N$  is an  $N$ -tuple of non-negative integers, and