

The Oblateness of an Axisymmetric Rotating Fluid

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§1. Introduction. In an earlier paper [5] we studied some quantitative aspects of the axisymmetric equilibrium figures of a rotating, self-gravitating fluid mass. In particular, we proved (essentially sharp) asymptotic estimates for the diameter of the fluid mass in terms of certain parameters characterizing the rotation rate of the fluid. In the present paper we establish a further asymptotic result concerning this model; roughly speaking, we show that the fluid, when rapidly rotating, must be contained in a “thin” slab about the equatorial plane. We now recall the model and summarize those results from [5] which are needed here.

Let $\rho(x)$, representing the density of the fluid, satisfy

$$(1.1) \quad \rho(x) \geq 0, \quad \rho(x) = \rho(r, z) = \rho(r, -z), \quad M = \int_{R^3} \rho(x) dx$$

where $x \in R^3$, $x = (r, \theta, z)$ in cylindrical coordinates (we also write $r = r(x)$). The density of a rotating, self-gravitating fluid mass is taken to be a function ρ which minimizes the total energy functional

$$(1.2) \quad E(\rho) = -\frac{1}{2} \int_{R^3} \int_{R^3} \frac{\rho(x)\rho(y)}{|x-y|} dx dy + \frac{1}{2} \int_{R^3} \frac{j^2(m(r(x)))}{r^2(x)} \rho(x) dx \\ + \beta K \int_{R^3} \rho^\gamma(x) dx$$

subject to the constraints (1.1); here, β and K are prescribed positive constants, $0 < \beta < 3$, and $\gamma = 1 + 1/\beta$. The angular momentum per unit mass, $j(m)$, is prescribed in terms of the mass fraction

$$m(r) = M^{-1} \int_{r(y) < r} \rho(y) dy.$$

We assume that $j(m)$ is increasing, $j(0) = 0$, and $j^2(m)$ is continuously differentiable on $0 \leq m \leq 1$.

The above pertains to a compressible fluid with the polytropic equation