

A Hölder Condition for the Local Time of the Brownian Sheet

ROBERT J. ADLER

1. Introduction. This paper is concerned with smoothness properties of the local times of the Brownian sheet. We let $W^{(N)}$, or simply W , denote the sheet, so that $W^{(N)}$ is a real valued Gaussian random field with zero mean and covariance

$$E\{W^{(N)}(s)W^{(N)}(t)\} = \prod_{i=1}^N \min(s_i, t_i),$$

where s and t belong to the parameter space \mathbf{R}_+^N of points in \mathbf{R}^N with all N components non-negative. If A and B denote, respectively, Borel sets in \mathbf{R}^1 and \mathbf{R}_+^N , then writing λ_N for N -dimensional Lebesgue measure we can define the occupation (bi)measure $\mu(A, B, \omega)$ for the Brownian sheet by

$$\mu(A, B, \omega) = \lambda_N\{t \in B: W^{(N)}(t, \omega) \in A\}.$$

From this measure, we define the local time function $L(x, B, \omega)$ to be any function L satisfying

$$\mu(A, B, \omega) = \int_A L(x, B, \omega) dx$$

for almost every ω . The local time can also be written as a function on $\mathbf{R}^1 \times \mathbf{R}_+^N$ by setting

$$L(x, t, \omega) = L\left(x, \prod_{i=1}^N [0, t_i], \omega\right)$$

for every $t \in \mathbf{R}_+^N$.

It is now well established that such a local time exists and, in its second form, possesses a version continuous on $\mathbf{R}^1 \times \mathbf{R}_+^N$. (Cairoli and Walsh [5], Theorem 6.4 for $N = 2$, and Davydov [6] for general N .) For certain purposes, it is often necessary to have more detailed knowledge of the properties of the Brownian sheet local time other than mere joint continuity. In particular, Hölder type bounds are necessary to study problems associated with the Hausdorff dimension of the level sets $\{t: W(t) = u\}$. That is, it is necessary