

# *On Density of Compact Operators in Reflexive Algebras*

CECELIA LAURIE

**1. Introduction.** Given a nonself-adjoint algebra of operators on a separable Hilbert space, a question of interest is when do the compact operators in the algebra determine the whole algebra. To shed light on this question for reflexive algebras we study the structure of the lattice of projections onto the subspaces left invariant by all the operators in the algebra. Various density properties are examined. These also imply that the algebra is determined by compact operators in the sense that the algebra equals the algebra of all operators leaving invariant the invariant subspaces of the compact operators in the algebra. We restrict our study to commutative lattices of projections on a separable Hilbert space and obtain information about the associated (reflexive) algebra of all operators leaving the given lattice invariant (i.e. we are restricting our attention to those reflexive algebras which contain a maximal abelian self-adjoint subalgebra).

Let  $\mathcal{H}$  be a separable Hilbert space. A commutative subspace lattice is a lattice of mutually commuting projections on  $\mathcal{H}$  containing 0 and  $I$  and closed in the strong operator topology. The simplest structure of such lattices is that of a *nest*, a linearly ordered lattice. In this case there is a density result of Erdős [3] which says that the finite rank operators in the associated nest algebra are strongly dense in the algebra. He actually obtains the stronger result that the finite rank operators in the unit ball of the algebra are strongly dense in the unit ball. This implies the existence of a strong approximate identity of finite rank operators in the nest algebra (i.e. a uniformly bounded net of finite rank operators converging strongly to the identity).

In this paper we examine the properties of strong and ultraweak density of Hilbert-Schmidt operators and the stronger property of the existence of a strong approximate identity of Hilbert-Schmidt operators in the associated algebra of a general commutative subspace lattice. One reason for examining the existence of an approximate identity of Hilbert Schmidt (or compact) operators in a (norm closed) algebra is that this implies that the algebra of compact perturbations of this algebra is norm closed (see Section 7).

A result of Arveson [1] states that any commutative subspace lattice on