

The Complete Solution to the (L^p, L^q) Mapping Problem for a Class of Oscillating Kernels

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§0. Introduction. Let K and f be two Lebesgue measurable functions on \mathbf{R}^n . Then the equation

$$K * f(x) = \int_{\mathbf{R}^n} K(x-t)f(t)dt$$

(whenever the integral exists) defines the convolution transform of K and f . In that case, we set $Tf = K * f$. In our work we are interested in solving the (L^p, L^q) mapping problem for K , that is to determine all pairs (p, q) for which

$$\|Tf\|_q \leq B\|f\|_p \quad \text{for } f \in L_0^\infty$$

(bounded functions with compact support) and where B is a positive constant independent of f .

In earlier papers [4], [8], [9], [12], [13] and [14] we discussed the (L^p, L^q) mapping problem for the kernels

$$(0) \quad K(t) = \frac{e^{i|t|^a}}{|t|^b}, \quad a \geq 0, \quad b < 1 \quad \text{and} \quad t \in \mathbf{R}.$$

In [8] and [9], we solved the (L^p, L^q) mapping problem for these kernels when $1 \leq p < q \leq \infty$. In the case where $p = q$, we were only able to prove that the endpoints satisfy weak restricted estimates. In fact, in the cases when $(a/2) + b \leq 1$ we solved the (L^p, L^q) mapping problem, see [9]. Here, we fill that gap by solving the (L^p, L^q) mapping problem for the kernels in (0) with $(a/2) + b > 1$ (see Theorem 5).

As an auxiliary result we prove here (see Theorem 4) that the kernels $K(t) = (1 + |t|)^{-1} e^{i|t|^a}$ for $a > 0$, $a \neq 1$ map H^1 into L^1 . In [5] and along with Stein in [6], Fefferman solved the (L^p, L^q) mapping problem for these kernels in (0) but with $a < 0$. Just recently C. Fefferman had pointed out to us that he also knew how to solve the (L^p, L^q) mapping problem when