

The Fixed Point Homomorphism for Maps of Period 2^k

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1. Introduction. The central problem in equivariant bordism is the classification of the various bordism types. One must both compute $\mathcal{N}_*(G)$ (in some sense) and also find determinant invariants. For $G = \{1\}$, the Stiefel-Whitney numbers solve this latter problem, and, in recent years, there have been several kinds of equivariant characteristic numbers proposed which have had partial success. In this paper we investigate the extent to which normal data determines the bordism class.

Specifically, let $G = \mathbb{Z}/2^k$. If L is any subgroup of G , there is the composite homomorphism

$$\chi_L: \mathcal{N}_*(G) \xrightarrow{r_L^G} \mathcal{N}_*(L) \xrightarrow{j} \mathcal{N}_*(L) \text{ (All, Proper),}$$

where r is induced by $L \subset G$ and j by the inclusion $(\text{All}, \phi) \subset (\text{All}, \text{Proper})$ of families of subgroups of L . Thus χ_L measures the extent to which G -bordism classes can be distinguished by the normal bundles to the fixed set of L . The *fixed-point homomorphism*

$$\chi = \bigoplus_{L \subset G} \chi_L: \mathcal{N}_*(G) \rightarrow \bigoplus_{L \subset G} \mathcal{N}_*(L) \text{ (All, Proper)}$$

was defined in [8]. For $k = 1$, χ is a monomorphism [4, page 73]. For $k \geq 2$, we shall show (see 3.4) that the kernel of χ is precisely the image of the extension homomorphism

$$e_G^H: \mathcal{N}_*(H) \rightarrow \mathcal{N}_*(G),$$

where $H = \mathbb{Z}/2^{k-1}$ is the maximal proper subgroup of G . Since [4, page 54] $r_H^G e_G^H$ is multiplication by two, and thus vanishes, it is obvious that $\text{Im } e_G^H$ is contained in $\text{Ker } \chi$; our result is that normal data detects as much of G -bordism as anyone could hope.

We mention here two other results of interest. Let d denote the homomorphism on G -bordism, of degree 1, defined by $d[M, \phi] = [S^1 \times_G M, 1 \times \phi]$. Then (see 3.1), the kernel of d is precisely the image of the restriction homomorphism from S^1 -bordism. Moreover, (see 5.1), a G -bordism class x is zero if and only if both $\chi(x)$ and $\chi(d(x))$ are zero.