

C-algebras Generated by Weighted Shifts II*

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Introduction. In this paper we study a family of simple C^* -algebras which are C^* -algebras generated by weighted shifts modulo the compact operators. It has been proved by the first author that if T is a weighted shift whose sequence of weights is an almost-periodic, non-periodic sequence of positive numbers which are bounded below then $C^*(T)$ contains the compact operators as a maximal ideal. In the first section of this paper we show that with such an operator T we can associate a countably infinite subgroup Λ of the unit circle Γ so that the crossed-product $C(\Gamma) \rtimes \Lambda$ of $C(\Gamma)$ with Λ (the group action being rotations of the circle through elements of Λ) is isomorphic to $C^*(T)/\mathcal{K}$. Conversely with every countably infinite subgroup Λ of Γ we can associate a weighted shift T so that $C^*(T)/\mathcal{K}$ is isomorphic to $C(\Gamma) \rtimes \Lambda$. In the second section we exploit this correspondence to prove the existence of projections in $C^*(T)/\mathcal{K}$ and discuss the questions of isomorphism. We conjecture that Λ determines the isomorphism class of $C^*(T)/\mathcal{K}$. In the particular case that Λ is a torsion group, we compute the K_0 and K_1 groups of $C(\Gamma) \rtimes \Lambda$ and show that in this case the isomorphism class is determined by Λ . We also show that $C(\Gamma) \rtimes \Lambda$ and hence $C^*(T)/\mathcal{K}$ can be imbedded in a UHF algebra whose “generalized integer” is determined by Λ .

Notation. H denotes a separable Hilbert space with an orthonormal basis $\{e_n\}_{n=0}^\infty$ and P_n denotes the orthogonal projection onto span of e_n . If $\{\alpha_n\}_{n=0}^\infty$ is a bounded sequence of positive numbers then T denotes the weighted shift with weights $\{\alpha_n\}_{n=0}^\infty$ i.e. $Te_n = \alpha_n e_{n+1}$ for all $n \geq 0$, D denotes the diagonal operator $De_n = \alpha_n e_n$ and S denotes the unweighted shift $Se_n = e_{n+1}$. $C^*(T)$ denotes the C^* -algebra generated by T and the identity operator I . If A is a C^* -algebra, G is a locally compact abelian group and θ is a continuous homomorphism of G into the group of automorphisms of A ($\text{Aut } A$) then $A \rtimes G$ denotes the crossed-product of G with A by the action θ of G . In particular if $A = C(\Gamma)$, Λ is any subgroup of Γ and Λ acts on $C(\Gamma)$ by rotation of Γ , then we denote the corresponding crossed-product by A_Λ . Note that A_Λ can be identified with the C^* -algebra of operators acting on $L^2(\Gamma)$