

Carleson Measures, H^1 Duality and Weighted BMO in Nonsmooth Domains

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Introduction. The results of this paper are organized in three sections. In each section, we indicate the background of the problems under consideration and we introduce the terminology used in their discussion.

The main result of Section 1 is as follows. Let D be a bounded, C^1 domain in \mathbf{R}^{n+1} with surface measure σ and harmonic measure ω evaluated at any fixed $P_0 \in D$ and let $K = d\omega/d\sigma$. Then, K and K^{-1} satisfy the Muckenhoupt condition $A_p(d\sigma)$ for all $p > 1$. This result is used frequently throughout the paper.

In Section 2 we take up the problem of characterizing the harmonic functions in D having boundary values in suitably weighted BMO classes on ∂D . The bounded domain D may be a Lipschitz or C^1 domain. As a matter of convenience, we shall assume that D is star-shaped with respect to some fixed P_0 , and evaluate the harmonic measure ω at P_0 . For any positive, ω -absolutely continuous measure ν on ∂D , we define $\text{BMO}_\nu(d\omega)$ as the set of all $f \in L^1(d\omega)$ such that

$$\sup_B \nu(B)^{-1} \int_B |f - f_{\omega(B)}| d\omega < \infty$$

where B are ‘‘surface balls’’ in ∂D and $f_{\omega(B)}$ is the integral average of f over B with respect to $d\omega$. Likewise, for any positive, σ -absolutely continuous ν on ∂D , $\text{BMO}_\nu(d\sigma)$ will denote all $f \in L^1(d\sigma)$ such that

$$\sup_B \nu(B)^{-1} \int_B |f - f_B| d\sigma < \infty$$

where $f_B = f_{\sigma(B)}$ now.

Corresponding to these weighted BMO classes, we consider harmonic functions u in D for which certain weighted Littlewood-Paley measures

$$d\mu(P) = |\nabla u(P)|^2 M(P) \rho(P) dP, \quad P \in D,$$

with $M(P) \geq 0$ and $\rho(P) = \text{dist}(P, \partial D)$, are ν -weighted Carleson measures;