

# *Boundary Behavior of Nonnegative Solutions of Elliptic Operators in Divergence Form*

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**Introduction.** This paper studies the existence of nontangential boundary limits for solutions of elliptic equations in divergence form with nonsmooth coefficients. Precisely we consider operators

$$L = \sum_{i,j=1}^n D_i(a_{ij}(X)D_j) \quad (D_i \equiv D_{x_i})$$

for which there exist positive numbers  $\lambda$  and  $\Lambda$  such that for all  $X$  and  $\xi \in R^n$

$$\lambda |\xi|^2 \leq \sum_{i,j=1}^n a_{ij}(X)\xi_i\xi_j \leq \Lambda |\xi|^2.$$

The coefficients  $a_{ij}(X)$  are assumed to be defined and measurable on  $R^n$  with  $a_{ij}(X) = a_{ji}(X)$ . We will let  $\mathcal{E}(\lambda, \Lambda)$  denote the class of operators  $L$  satisfying the above conditions for a fixed  $\lambda$  and  $\Lambda$ .

By a solution,  $u$ , of  $Lu = 0$  in a domain,  $D$ , of  $R^n$  we mean a function  $u \in L^2_{loc}(D)$  whose partial derivatives,  $D_i u$ , in the distribution sense, belong to  $L^2_{loc}(D)$  and such that

$$\sum_{i,j=1}^n \int_D a_{ij}(X) D_j u(X) D_i \varphi(X) dX = 0$$

for all  $\varphi \in C^\infty_0(D)$ , the class of  $C^\infty$  functions with compact support in  $D$ . A fundamental result of DiGiorgi and Nash ([4], [10] and Theorem 0.7 below) is that any solution of  $Lu = 0$  in  $D$  is in fact Hölder continuous on any compact subdomain of  $D$ .

Our main result states that any *nonnegative* solution of  $Lu = 0$  in the unit ball ( $L \in \mathcal{E}(\lambda, \Lambda)$ ) has nontangential limits at each point on the boundary which does not belong to a set of “ $L$ -harmonic” measure zero. The notion of “ $L$ -harmonic” measure is defined using the same construction as for the