

A Sharp Estimate for a Multilinear Singular Integral in \mathbf{R}^n

JONATHAN COHEN

§0. Introduction. The purpose of this paper is to prove a good λ inequality for certain multilinear singular integrals in \mathbf{R}^n . The operators considered here are the maximal and limiting operators of the singular integral

$$C_\epsilon(\nabla A, f)(x) = \int_{|x-y|>\epsilon} P_2(A; x, y) \frac{\Omega(x-y)}{|x-y|^{n+1}} f(y) dy$$

where $P_2(A; x, y) = A(x) - A(y) - \nabla A(y) \cdot (x - y)$ is the second order Taylor series remainder of A , Ω satisfies certain homogeneity, smoothness and symmetry conditions, $\nabla A \in \text{BMO}$ and $f \in L^p(\mathbf{R}^n)$. The maximal function $C_*(\nabla A, f)(x) = \sup_{\epsilon>0} |C_\epsilon(\nabla A, f)(x)|$ is shown to be controlled by a “sharp function” of ∇A and a Hardy-Littlewood maximal function of f .

The operators considered in this paper are related to the commutators studied by Calderón in [2]. An extension of Calderón’s result to \mathbf{R}^n by Baishansky and Coifman [1] is an essential part of the proof of the main result of this paper.

The methods in this paper are similar to those used by Coifman and Meyer in studying the second commutator on the line and by Coifman, Rochberg and Weiss for studying the first commutator in \mathbf{R}^n . However, the higher order Taylor series remainder occurring in the operators considered in this paper require a modification of the procedure.

In studying commutators of singular integrals, Coifman, Rochberg and Weiss [5] replace the term $b(y) - b(x)$ by $b(y) - m_{Q_x}(b)$ where $m_{Q_x}(b)$ is the average of b over the cube Q_x centered at x . They obtain an \tilde{L}^p estimate for this new operator in terms of the BMO norm of b which is independent of the choice of the cube Q_x . By shrinking the cube one gets the estimate for the commutator.

This procedure doesn’t work in the Taylor series remainder case. Instead we use the fact that $P_2(A; x, y) = P_2(A_Q; x, y)$ where $A_Q(x) = A(x) - \sum_{j=1}^n m_Q(D_j A) x_j$ and the fact that $(A_Q(x) - A_Q(y))/|x - y|$ can be estimated in terms of a “sharp” function of ∇A .