

Minimal Surfaces of Revolution in H^3 and Their Global Stability

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Dedicated to Professor Kisuke Tsuchida on his 60th birthday

1. Introduction. There are many complete regular minimal surfaces in Euclidean 3-space E^3 , but there have been few results on such surfaces in the simply connected, complete hyperbolic 3-space H^3 of constant sectional curvature -1 .

The purpose of this note is to construct complete regular minimal surfaces of revolution in H^3 and to show that some of them are globally stable, complete minimal surfaces. This fact contrasts with the result of do Carmo and Peng [2].

In Section 2, we review the hyperbolic n -space H^n as the standard model of a simply connected, complete Riemannian n -manifold of constant sectional curvature -1 and discuss the properties of the generating curve of a minimal surface of revolution in H^3 . In Section 3 we linearize and solve a system of non-linear differential equations of second order which arises in Section 2. In Section 4, we show that some of the minimal surfaces of revolution in H^3 obtained in Section 3 are globally stable.

2. Preliminaries. In the vector space R^{n+1} with the natural basis e_1, \dots, e_{n+1} , we consider a non-degenerate symmetric bilinear form $\langle \cdot, \cdot \rangle$ defined by

$$\langle x, y \rangle = -x^1 y^1 + \sum_{j=2}^{n+1} x^j y^j, \quad x, y \in R^{n+1}.$$

The hyperbolic n -space H^n is the simply connected hypersurface of R^{n+1} defined by $H^n = \{x \in R^{n+1} : \langle x, x \rangle = -1, x^1 \geq 1\}$. The tangent space $T_x(H^n)$ at $x \in H^n$ is given, through the identification by parallel displacement in R^{n+1} , by the subset of all vectors $u \in R^{n+1}$ such that $\langle u, x \rangle = 0$. The restriction of $\langle \cdot, \cdot \rangle$ to $T_x(H^n)$ is positive definite. Then it is known that the form $\langle \cdot, \cdot \rangle$ restricted to the tangent space at each point of H^n gives rise to a complete analytic Riemannian metric on H^n whose sectional curvature is the constant -1 (see [3] for detailed discussion).

Let $\gamma(s) = (x(s), y(s), z(s))$, $s \in I$, by any C^2 -curve in the hyperbolic plane