On the Maximum Principle for Pseudoparabolic Equations

EMMANUELE DI BENEDETTO & MICHEL PIERRE

Introduction

Let A be an m-accretive operator in a Banach space X; it is well-known that if its resolvents J_{λ} satisfy:

$$\forall \lambda > 0 \quad J_{\lambda} C \subset C,$$

for a given closed convex set C of X, then the solution of:

(P)
$$\frac{du}{dt} + Au \ni 0, \qquad u(0) = u_0,$$

satisfies:

$$(2) u_0 \in C \Rightarrow \forall t \quad u(t) \in C.$$

Here, we study the same problem for the associated pseudoparabolic equations:

(PP)
$$\frac{d}{dt}(u + \lambda A u) + A u \ni 0, \qquad u(0) = u_0 \qquad (\lambda > 0).$$

or, more generally, if A(t) is a family of *m*-accretive operators satisfying (1):

$$(PP)_{t} \qquad \frac{d}{dt}(u + \lambda A(t)u) + A(t)u \ni 0, \qquad u(0) = u_{0}.$$

In order to exhibit a concrete situation, we remark that equation (PP)_t contains as a particular case, the following problem:

(E)
$$\begin{cases} \frac{\partial}{\partial t} (u - \lambda \Delta u) - \Delta u = 0 & \text{in } \Omega \times [0, T[\\ u(t)|_{\partial \Omega} = g(t), & u(0) = u_0, \end{cases}$$

821

Indiana University Mathematics Journal ©, Vol. 30, No. 6 (1981)