

A Class of Infinite-Dimensional Diffusions Occurring in Population Genetics

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1. Introduction. Let S be a countable set. The aim here is to obtain sufficient conditions for the existence of a Feller-continuous diffusion process $X(\cdot)$ in the simplex

$$(1.1) \quad \Delta = \left\{ x = (x_i)_{i \in S} \in [0, 1]^S : \sum_{i \in S} x_i = 1 \right\}$$

with generator of the form

$$(1.2) \quad \Omega = \frac{1}{2} \sum_{i, j \in S} x_i (\delta_{ij} - x_j) \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i \in S} b_i(x) \frac{\partial}{\partial x_i},$$

where the domain $\mathcal{D}(\Omega)$ of Ω is the set of all twice continuously differentiable functions on Δ depending on only finitely many coordinates. Such diffusions occur as approximations to discrete stochastic models for molecular evolution in a finite population in which the set of possible alleles or “types” is indexed by S . In this context, $X_i(t)$ is interpreted as the proportion of the population that is of type i at time t .

A well-known example is the charge-state model (or ladder model) of Ohta and Kimura [10], in which case $S = \mathbb{Z}^1$ and

$$(1.3) \quad b_i(x) = \frac{1}{2} \theta (x_{i-1} + x_{i+1}) - \theta x_i$$

for some constant $\theta > 0$.

Our approach is to adapt a technique of Liggett [7], which he used to obtain sufficient conditions for the existence of infinite particle systems. We recall Liggett’s result in the special case of the so-called spin-flip system, which is a Markov process in the configuration space

$$(1.4) \quad E = \{-1, 1\}^S$$

with generator of the form

$$(1.5) \quad \mathcal{L} = \sum_{i \in S} c_i(\eta) \Delta_i,$$