

Integral Operators and Changes of Density

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1. Introduction. In [10] W. B. Johnson and L. Jones observe that after an appropriate change of density every continuous linear operator $T : L_1(\Omega, \mu) \rightarrow L_1(\Omega, \mu)$ is an L_q -operator for all $1 \leq q \leq \infty$, i.e. there is a $g \in L_1(\Omega, \mu)$ such that $\rho T \rho^{-1} : L_q(\Omega, \mu g d\mu) \rightarrow L_q(\Omega, \mu g d\mu)$ for all $1 \leq q \leq \infty$ where ρ is just a multiplication operator $\rho(f) = f \cdot g^{-1}$. In this paper I want to point out that a result of this kind is typical for positive operators and regular operators (i.e. differences of positive operators) and that it has interesting applications to integral operators. More precisely: By choosing an appropriate density we show in Section 2 that a linear operator $T : L_p(\Omega, \mu) \rightarrow L_p(\Omega, \mu)$ is regular if and only if there is a positive isometry J of $L_p(\Omega, \mu)$ such that $JTJ^{-1} : L_q(\Omega, \mu) \rightarrow L_q(\Omega, \mu)$ for all $1 \leq q \leq \infty$. There is also a somewhat weaker Banach lattice version of this result: If T is regular in a Banach lattice X then (under mild assumptions on X) there is a representation \tilde{X} of X as a space of measurable functions on some measure space (Ω, μ) so that in this representation $\tilde{T} : L_q(\Omega, \mu) \rightarrow L_q(\Omega, \mu)$ for all $1 \leq q \leq \infty$.

The remainder of the paper gives applications to integral operators, i.e. operators K defined on an ideal X of measurable functions on (Ω, μ) with a representation

$$Kf(s) = \int_{\Omega} K(s, t)f(t)d\mu(t), \text{ a.e.}$$

where we only assume that k is measurable on $\Omega \times \Omega$ and $k(s, \cdot)f(\cdot)$ is integrable for μ -almost all $s \in \Omega$ and all $f \in X$. Generally speaking, one can use the change of density technique in order to reduce problems in the general situation to the better known case where $X = L_{\infty}(\Omega, \mu)$ or $X = L_1(\Omega, \mu)$. In this way we give new proofs to characterizations of integral operators due to A. Bukhvalov [3] and W. Schachermayer [18] by reducing their results to a classical theorem of Dunford and Pettis. This provides a more analytical proof of the theorem of Bukhvalov who had used the framework of lattice theory in his original paper, and we get a generalization of Schachermayer's result who had restricted himself to the case $X = L_p(\Omega, \mu)$. (See Section 3.)

In Sections 4 and 5 we are concerned with compactness properties of integral operators, the basic result being that an integral operator $K : L_p \rightarrow L_p$ has "large" compact restrictions $\chi_E K$. This property actually characterizes the norm closure of the set of integral operators in $B(L_p(\Omega, \mu))$ (see 4.4.) and this answers a question