

Probabilities of Extinction of Multiplicative Measure Diffusion Processes with Absorbing Boundary

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1. Introduction. Many authors ([2], [4], [6], [9], [18], [21], [22]) have used the notion of measure-valued processes to model the evolutions of geographically distributed populations. A basic model of this type is the multiplicative measure diffusion process or the continuous state branching process ([2], [4], [11], [15], [18], [22]) which arise as the high density approximations of branching Markov processes ([10], [18], [21], [24], [26]) or age-dependent spatially distributed branching processes ([22]). A branching Markov process which models a reproducing population in the absence of interaction effects is a branching process in which each particle has a “location” in some space D (usually a subset of Euclidean n -space \mathbf{R}^n) and moves, up to its time of branching, as a Markov process with state space D ([10], [18], [25], [11], [13], [22]).

The asymptotic behavior of the number of individuals of a branching Markov process has been studied by many authors ([12], [21], [24], [25]) and various interesting limit theorems have been obtained. However, as far as asymptotic behavior is concerned, less is known about the multiplicative measure diffusion processes ([3], [5], [23]). It is the purpose of this paper to study the probabilities of extinction of multiplicative measure-valued diffusions on a bounded domain D in \mathbf{R}^n with absorbing boundary.

In the next section, we shall review the basic definition and results of a multiplicative stochastic measure diffusion process and introduce the equations that control the process. We will consider a measure-valued diffusion process defined on a bounded domain $D \subseteq \mathbf{R}^n$ with boundary ∂D as an absorbing barrier. In this case, the process is determined by the following semilinear parabolic equation

$$\begin{aligned} \frac{\partial u(x,t)}{\partial t} &= \beta \Delta u + \alpha u(1-u) && \text{on } D \times (0, \infty), \\ (1.1) \quad u(x,t) &= 0 && \text{on } \partial D, \\ u(x,0) &= f(x) > 0, \end{aligned}$$