

# Singularities of Quasiconformal Deformations in $R^n$

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## 1. Introduction

**1.1.** Let  $G$  be an open set in the  $n$ -dimensional euclidean space  $R^n$ ,  $n \geq 2$ . A continuous mapping  $f : G \rightarrow R^n$  is called a quasiconformal deformation, abbreviated qc-deformation or qcd (= quasiconformally deforming), if it has locally integrable first order distributional derivatives  $D_i f$ ,  $i = 1, 2, \dots, n$ , in  $G$  and  $\|Sf\|_\infty = \text{ess sup}_{x \in G} \|Sf(x)\| < \infty$ . Here  $S$  is Ahlfors' differential operator

$$(1.2) \quad Sf(x) = \frac{1}{2} \left[ Df(x) + \widetilde{Df(x)} \right] - \frac{1}{n} \text{tr} (Df(x))I$$

where  $Df(x) = (D_j f_i(x))$  is the Jacobian matrix of  $f$  at  $x$ ,  $I$  is the unit matrix,  $\widetilde{A}$  denotes the transpose and  $\text{tr} (A)$  the trace of an  $n \times n$  - matrix  $A$ . The norm  $\|A\|$  is the supremum norm  $\|A\| = \sup_{|x|=1} |Ax|$ ,  $x \in R^n$ . If  $\|Sf\|_\infty \leq k < \infty$ ,  $f$  is called a  $k$ -qc-deformation, or in brief,  $k$ -qcd.

If  $n = 2$ ,  $Sf(z)$  corresponds to the Cauchy-Riemann differential operator  $\partial f / \partial \bar{z} = (1/2)(\partial f / \partial x + i \partial f / \partial y)$ ,  $z = x + iy \in C$ ; namely,  $Sf(z)u = (\partial f / \partial \bar{z})(z)\bar{u}$  where  $\bar{u}$  is the complex conjugate of  $u \in C$ . Therefore,  $Sf = 0$  is the Cauchy-Riemann equation for  $n = 2$ , and analytic mappings form a subclass of qc-deformations in the plane.

In general, a qc-deformation  $f$  is called trivial, if  $Sf = 0$ . Due to [A1, Proposition 1] there are rather few trivial deformations in the space; namely, if  $f : G \rightarrow R^n$  is a trivial deformation,  $G$  is connected and  $n \geq 3$ , then

$$(1.3) \quad f(x) = a + Bx + 2(c \cdot x)x - |x|^2 c, \quad x \in G,$$

where  $a, c \in R^n$ ,  $B : R^n \rightarrow R^n$  is a linear mapping with  $SB = 0$  and  $c \cdot x$  denotes the scalar product in  $R^n$ .

The main purpose of this paper is to prove that a spatial qc-deformation cannot have isolated singularities. In other words, we show that if  $f : G \setminus \{x_0\} \rightarrow R^n$  is  $k$ -qcd,  $x_0 \in G$  and  $G$  is open in  $R^n$ ,  $n \geq 3$ , then  $f$  has a  $k$ -qcd extension to  $G$ . This result turns out to be equivalent to the following growth result: if  $f : R^n \rightarrow R^n$  is