Axially Symmetric Infinite Cavities

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Introduction. The mathematical literature of a cavity past a moving nose (or barrier) goes back to Helmholtz (1868) and Kirchoff (1869). A detailed account of the theory can be found in the monographs of Birkhoff and Zarantonello [1] and Gilbarg [5]; see also a more recent survey by Wu [12]. For two-dimensional flows the hodograph method has been used in conjunction with conformal mappings in order to compute solutions for specific barriers; this method also provides a tool by which existence theorems can be reduced to more tractable problems of solving certain nonlinear integral equations. However, these two-dimensional methods do not extend to three-dimensional cavitational flows.

For three dimensions, the only general existence theorem is that of Garabedian, Lewy and Schiffer [3] which considers an axially symmetric inviscid ideal flow for the Riabouchinsky model. In this model one introduces, behind the nose N, a tail T which is the reflection of N with respect to a vertical plane P (see Figure 1), and then establishes the existence of a cavity "between" N and T; the cavity is the shaded set and the free boundary is denoted by Γ (it is analytic).

At the end of [3] the authors let P move rightward toward infinity in order to obtain an infinite cavity (Figure 2).

Setting Γ_P = the free boundary corresponding to a particular choice of P, they prove that $\Gamma = \lim \Gamma_P$ is analytic and that Γ contains a sequence of points tending to infinity and lying within a certain angle about the axis of symmetry.

The purpose of the present paper is to further study this infinite cavity and to establish some of the basic physical properties. Let us denote the equation of the free boundary (in Figure 2) by $y = f(x)$ (y = distance from the axis of symmetry) and the stream function by $\psi(x,y)$. We shall establish the following results:

The magnitude of the velocity is equal to 1 on the free boundary, that is,

$$\frac{1}{y}\frac{\partial \psi}{\partial \nu} = 1 \qquad \text{on } \Gamma \quad (\nu \text{ the inward normal}); \tag{0.1}$$

the velocity vector tends to (1,0) at infinity, that is,

$$\frac{1}{y}\frac{\partial \psi}{\partial x} \to 0, \qquad \frac{1}{y}\frac{\partial \psi}{\partial y} \to 1 \qquad \text{if } x^2 + y^2 \to \infty; \tag{0.2}$$

the free boundary becomes horizontal at infinity, that is,

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