

# A Counter Example for Weiner's Open Question

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**0. Introduction.** Let  $f : M \rightarrow M'$  be an isometric immersion of an orientable surface  $M$  in an  $n$ -dimensional Riemannian manifold  $M'$ . We denote by  $H$  and  $R'$  the mean curvature of  $f$  and the sectional curvature of  $M'$  with respect to the tangent space of  $M$ . Then we define  $\tau(f)$  by

$$(1) \quad \tau(f) = \int_M (H^2 + R') *l,$$

where  $*l$  is the volume element of  $M$ . In [1], B. Y. Chen shows that  $\tau$  is an invariant under conformal changes of the metric  $\langle \cdot, \cdot \rangle$  of  $M'$ . The variation of  $\tau$  is calculated in [2]. When  $M'$  is a space form,  $f$  is a stationary point of  $\tau$  if and only if

$$(2) \quad \Delta \mathbf{H} - 2H^2 \mathbf{H} + A(\mathbf{H}) = 0,$$

where  $\Delta$  is the Laplacian with respect to the normal connection,  $\mathbf{H}$  is the mean curvature vector and  $A$  is a field of endomorphisms of normal spaces defined in terms of the second fundamental form  $\sigma$  of the immersion  $f$  and an orthonormal frame  $e_1, e_2$  of  $M$  by

$$A(N) = \sum_{i,j=1}^2 \langle N, \sigma(e_i, e_j) \rangle \sigma(e_i, e_j).$$

For stationary immersions in an  $n$ -dimensional unit sphere  $S^n$ , Weiner gives the following open question in [2].

**Question.** *Are the only closed orientable immersed surfaces in  $S^n$  whose centroids are 0 and which satisfy*

$$\Delta \mathbf{H} - 2H^2 \mathbf{H} + A(\mathbf{H}) = 0$$

*minimal surfaces in  $S^n$ ?*

The centroid of an immersion  $f : M \rightarrow S^n \subset R^{n+1}$  is the vector of  $R^{n+1}$  defined by