

# *Existence and Multiplicity Theorems for Semilinear Elliptic Equations with Nonlinear Boundary Conditions*

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**1. Notation and statement of the results.** In this paper we are looking for solutions of the semilinear elliptic boundary value problem

$$(BP) \quad \begin{cases} Au = f(x, u, Du) & \text{in } \Omega \\ Bu = g(x, u) & \text{on } \partial\Omega. \end{cases}$$

Here,  $\Omega$  will denote a bounded domain of class  $C^2$  in  $\mathbf{R}^N$ ,  $N \geq 2$ , i.e. the boundary  $\partial\Omega$  is a  $C^2$ -manifold of dimension  $N - 1$  such that  $\Omega$  lies locally on one side of  $\partial\Omega$ . By  $A$  we mean the differential operator

$$(1.1) \quad Au = - \sum_{j,k=1}^N a_{jk}(x) D_j D_k u,$$

where  $a_{jk} = a_{kj} \in C(\bar{\Omega})$  and  $A$  is uniformly strongly elliptic, i.e. there exists some constant  $\mu > 0$  with  $\sum_{j,k=1}^N a_{jk}(x) \xi^j \xi^k \geq \mu |\xi|^2$  for all  $x \in \bar{\Omega}$  and  $\xi \in \mathbf{R}^N$ .

The boundary operator  $B$  is defined by

$$(1.2) \quad Bu = \frac{\partial u}{\partial \beta},$$

where  $\beta \in C^1(\partial\Omega, \mathbf{R}^N)$  is an outward pointing, nowhere tangent vector field on  $\partial\Omega$ .

For the nonlinearities  $f$  and  $g$  we suppose throughout this paper

$$(f1) \quad \begin{cases} f: \bar{\Omega} \times \mathbf{R} \times \mathbf{R}^N \rightarrow \mathbf{R} \text{ is continuous and } \partial f / \partial \xi, \partial f / \partial \eta \\ \text{exist and are continuous, too. } ((x, \xi, \eta) \text{ denotes a} \\ \text{generic point of } \bar{\Omega} \times \mathbf{R} \times \mathbf{R}^N.) \end{cases}$$

$$(f2) \quad \begin{cases} \text{There is an increasing function } c: \mathbf{R}_+ \rightarrow \mathbf{R}_+ \text{ such that} \\ |f(x, \xi, \eta)| \leq c(|\xi|) (1 + |\eta|^2) \text{ for all } (x, \xi, \eta) \in \bar{\Omega} \times \mathbf{R} \times \mathbf{R}^N. \end{cases}$$

$$(g) \quad g: \partial\Omega \times \mathbf{R} \rightarrow \mathbf{R} \text{ is locally Lipschitz continuous.}$$