

A Semilinear Reaction-Diffusion Prey-Predator System with Nonlinear Coupled Boundary Conditions: Equilibrium and Stability

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1. Introduction and preliminaries. The reaction-diffusion equation for prey-predator interaction:

$$(1.1) \quad \begin{aligned} \frac{\partial u_1}{\partial t} &= \sigma_1 \Delta u_1 + u_1[a + f_1(u_1, u_2)] \\ \frac{\partial u_2}{\partial t} &= \sigma_2 \Delta u_2 + u_2[-r + f_2(u_1, u_2)] \end{aligned}$$

will be studied, where $\Delta = \sum_{i=1}^n \partial^2 / \partial x_i^2$, a, r, σ_1, σ_2 are positive constants, $f_i : \mathbb{R}^2 \rightarrow \mathbb{R}$ have Hölder continuous partial derivatives up to a second order in compact sets, $i = 1, 2$. Further, we assume that

$$(1.2) \quad \begin{aligned} f_i(0,0) &= 0, \quad i = 1, 2; \\ \frac{\partial f_1}{\partial u_1} &< 0, \quad \frac{\partial f_1}{\partial u_2} < 0, \quad \frac{\partial f_2}{\partial u_1} > 0, \quad \frac{\partial f_2}{\partial u_2} < 0 \end{aligned}$$

for (u_1, u_2) in the first open quadrant, and for each $m \geq 0$

$$(1.3) \quad \lim_{u_2 \rightarrow +\infty} f_2(m, u_2) = -\infty, \quad \lim_{u_1 \rightarrow +\infty} f_2(u_1, m) = +\infty, \quad \lim_{u_1 \rightarrow +\infty} f_1(u_1, 0) = -\infty.$$

The functions $u_1(x, t)$, $u_2(x, t)$ respectively represent the density of prey and predator at position $x = (x_1, \dots, x_n)$ and time $t \geq 0$. The parameters σ_1, σ_2 are diffusion rates; a and r are growth and mortality rates of the prey and predator in the event that no interaction involves between them. f_1, f_2 describe further interactions, and (1.2), (1.3) are general assumptions which include the modified Volterra-Lotka model with crowding effects. The problem of existence of solu-