

Unbounded Parametric Surfaces of Prescribed Mean Curvature

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In this paper, I show the existence of a set Ω which minimizes the energy functional

$$\mathcal{E}(\Omega) = \int |D\chi_\Omega| + \int_\Omega H(x)dx$$

with a given obstacle Q over compact perturbations. $\int |D\chi_\Omega|$ is the perimeter of Ω in the sense of De Giorgi. Here $H(x)$ is assumed to be in $L^1_{\text{loc}}(\mathbf{R}^n)$ and to be non-negative. Q need not be bounded, but it will be assumed to be a Caccioppoli set. If H is continuous, if Ω minimizes \mathcal{E} over compact perturbations, and if $\partial\Omega$ is C^2 , then at a point $x \in \partial\Omega$, $\partial\Omega$ has mean curvature $H(x)$. Massari [3] has extended the notion of mean curvature to say that the boundary of a set Ω , which minimizes \mathcal{E} over compact perturbations, has prescribed mean curvature $H(x)$ without assuming continuity of $H(x)$ or smoothness of $\partial\Omega$.

The results from the theory of functions of bounded variation necessary for this paper may be found (for the most part) in Giusti [2, Chapter 1].

1. Definition. Let E be a Borel set and χ_E its characteristic function. Let A be an open set. Then the *perimeter of E in A* , which I will write as $\int_A |D\chi_E|$, is defined as:

$$\int_A |D\chi_E| = \sup \left\{ \int_E \operatorname{div} g \, dx : g \in C^1_0(A; \mathbf{R}^n), |g| \leq 1 \right\}.$$

This can be extended to define $\int_B |D\chi_E|$ for any set B by:

$$\int_B |D\chi_E| = \inf \left\{ \int_A |D\chi_E| : A \text{ is open, } B \subseteq A \right\}.$$

It can be shown that if E is a fixed Borel set, then $\int_B |D\chi_E|$ is an outer measure on the set B , and that all Borel sets B are measurable with respect to this outer measure. If ∂E is C^2 , then $\int_{\mathbf{R}^n} |D\chi_E|$ is the surface area of ∂E .

2. Remark. If $\mu(E\Delta\tilde{E}) = 0$, where μ is Lebesgue measure and $E\Delta\tilde{E} = (E - \tilde{E}) \cup (\tilde{E} - E)$ is the symmetric difference, then