

Torus Actions on 5- and 6-Manifolds

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Suppose M is a closed, orientable 6-manifold admitting a locally smooth, effective action of T^3 with $M^* \approx D^3$. Suppose the interior points of M^* represent principal orbits and that points on ∂M^* correspond to T^1 - or T^2 -stability groups. In this case, circles on ∂M^* will correspond to orbits with T^2 -stability groups and the remaining points on ∂M^* will correspond to T^1 -stability groups.

In [4], simply connected manifolds admitting such actions were classified under the additional hypothesis that $H_*(M)$ is torsion free and $w_2(M) = 0$. However, some of the proofs in [4] are not valid.

Here we give different proofs of more general results that are valid when $H_*(M)$ has torsion. Furthermore, results in [3], generalizing those in [10], enable us to eliminate the hypothesis that $w_2(M) = 0$.

Results in dimension five are very similar to those in dimension six. We prove here a classification theorem for smooth action of T^2 on closed, orientable, smooth 5-manifolds.

If a group G acts on a space M , we let M^* represent the orbit space and $p : M \rightarrow M^*$ the orbit map. If $X \subset M$, we let $X^* = p(X)$ and for $X^* \subset M^*$ we let $X = p^{-1}(X^*)$. We will use F to denote the fixed point set, E the union of exceptional orbits and P the union of principal orbits.

Let $q : R^n \rightarrow T^n$ be the universal covering projection defined by $q(x_1, \dots, x_n) = (\exp 2\pi x_1 i, \dots, \exp 2\pi x_n i)$. Suppose T is a circle subgroup of T^n . Then each component of $q^{-1}(T)$ is a line containing at least two lattice points in R^n . Hence it is natural to parameterize a circle subgroup of T^n by $G(a_1, \dots, a_n) = \{(x_1, \dots, x_n) : x_1 = a_1 t, \dots, x_n = a_n t, 0 \leq t < 1\} \pmod{\mathbf{Z}^n}$. Here a_1, \dots, a_n are relatively prime integers. For example, we do not allow $G(0, 2, 0)$ etc, since $\gcd(0, 2, 0) = 2 > 1$.

By the determinant of circle stability groups $G(a_1, b_1, c_1)$, $G(a_2, b_2, c_2)$, and $G(a_3, b_3, c_3)$ we mean the determinant of three vectors (a_1, b_1, c_1) , (a_2, b_2, c_2) , and (a_3, b_3, c_3) . Suppose M_1 and M_2 are G -manifolds. Then a diffeomorphism of M_1 onto M_2 is (weakly) equivariant if there is an automorphism α of G such that $f(g \cdot x) = \alpha(g) \cdot f(x)$ for $g \in G$ and $x \in M_1$.

Unless otherwise stated we shall always use \mathbf{Z} as coefficients in (co)homology. The first Pontrjagin class of M will be denoted by $p_1(M)$. Finally, we have the trilinear form

$$\mu : H^2(M) \times H^2(M) \times H^2(M) \rightarrow \mathbf{Z}$$