

*On the Elliptic Equation*  
 $\Delta u + K(x)u^{(n+2)/(n-2)} = 0,$   
*its Generalizations, and*  
*Applications in Geometry*

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**§1. Introduction and statement of main results**

In this paper we consider the existence problem of positive solutions of the elliptic equation

$$(1.1) \quad \Delta u + K(x)u^{(n+2)/(n-2)} = 0$$

in  $\mathbf{R}^n$ , where  $\Delta = \sum_1^n \partial^2/\partial x_j^2$ , and  $n \geq 3$ . This problem has its root in Riemannian geometry.

Let  $(M, g)$  be a Riemannian manifold of dimension  $\geq 3$  and  $K$  be a given function on  $M$ . One may ask the question: can we find a new metric  $g_1$  on  $M$  such that  $K$  is the scalar curvature of  $g_1$  and  $g_1$  is *conformal* to  $g$  (i.e.  $g_1 = u^{4/(n-2)}g$ , for some  $u > 0$  on  $M$ )? This is equivalent to the problem of finding positive solutions of the equation

$$(1.2) \quad \frac{4(n-1)}{n-2} \Delta u - ku + Ku^{(n+2)/(n-2)} = 0$$

where  $\Delta, k$  are the Laplacian and scalar curvature in the  $g$  metric, respectively. In the case where  $M$  is compact, this problem has been considered by many authors, we refer to [9] for further details and references. In the case where  $M = \mathbf{R}^n$  and  $g$  is the usual metric, then  $k = 0$  and equation (1.2) reduces to

$$(1.3) \quad \frac{4(n-1)}{n-2} \Delta u + K(x)u^{(n+2)/(n-2)} = 0.$$

Equation (1.3) differs from (1.1) only by a constant which can be easily absorbed because of the different degrees of homogeneity of the two terms  $\Delta u$  and  $Ku^{(n+2)/(n-2)}$ . Therefore, throughout this paper, we shall consider equation (1.1) instead of (1.3) for the sake of convenience.