

Maximal Invariant Spaces of Analytic Functions

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Let $H(\mathcal{D})$ denote the topological vector space of holomorphic functions on a bounded symmetric domain \mathcal{D} in \mathbf{C}^m . The topology on $H(\mathcal{D})$ is the topology of uniform convergence on compact sets. $\text{Aut}(\mathcal{D})$ denotes the group of biholomorphic mappings of \mathcal{D} onto \mathcal{D} . For the case of irreducible domains \mathcal{D} , the main result is the following one.

Theorem 0.1. *Let \mathcal{D} be irreducible. If $L \neq 0$ is a continuous linear functional on $H(\mathcal{D})$ and $f \in H(\mathcal{D})$ satisfies*

$$\sup \{|L(f \circ \phi)| : \phi \in \text{Aut}(\mathcal{D})\} = M < \infty,$$

then f is a Bloch function [7] on \mathcal{D} and the Bloch seminorm $\|f\|_{\mathcal{B}}$ of f is at most $c_L M$ where c_L is a constant depending only on L .

Bloch functions on \mathcal{D} will be defined later. We will also give a corresponding theorem for the reducible case. The following is an immediate consequence of Theorem 0.1.

Corollary 0.2. *Let \mathcal{D} be irreducible. Then the only $\text{Aut}(\mathcal{D})$ -invariant closed linear subspaces of $H(\mathcal{D})$ are $\{0\}$, the constant functions and $H(\mathcal{D})$.*

Now let $(X, \|\cdot\|_X)$ denote a linear space X of analytic functions on \mathcal{D} equipped with a seminorm $\|\cdot\|_X$. The space $(X, \|\cdot\|_X)$ is called $\text{Aut}(\mathcal{D})$ -invariant (or just invariant) if, whenever $f \in X$ and $\phi \in \text{Aut}(\mathcal{D})$, then $f \circ \phi \in X$ and $\|f \circ \phi\|_X = \|f\|_X$. A non-zero linear functional on X is called ‘‘decent’’ if it is continuous with respect to $\|\cdot\|_X$ and it extends to a continuous linear functional on $H(\mathcal{D})$. The following is actually equivalent to Theorem 0.1.

Theorem 0.3. *Let \mathcal{D} be irreducible. Let $(X, \|\cdot\|_X)$ be an $\text{Aut}(\mathcal{D})$ -invariant seminormed space of analytic functions on \mathcal{D} . Suppose $(X, \|\cdot\|_X)$ possesses one (non-zero) decent linear functional. Then X is contained in the set $\mathcal{B}(\mathcal{D})$ of Bloch functions on \mathcal{D} and the inclusion mapping from $(X, \|\cdot\|_X)$ to $(\mathcal{B}(\mathcal{D}), \|\cdot\|_{\mathcal{B}})$ is continuous.*

For the case $\mathcal{D} = D =$ the unit disc in \mathbf{C} , Theorem 0.3 and Corollary 0.2 were proved in [5]. Examples were given there of well-known spaces $(X, \|\cdot\|_X)$ satisfying the hypotheses of Theorem 0.3 and also of a space failing to satisfy both