

Multiple Periodic Solutions for an Asymptotically Linear Wave Equation

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In this paper we study the multiple periodic solutions of the following asymptotically linear wave equation:

$$(0.1) \quad \begin{cases} u_{tt} - u_{xx} + g(x,t;u) = 0, & (x,t) \in Q = (0,\pi) \times (0,2\pi) \\ u(0,t) = u(\pi,t) = 0, & t \in [0,2\pi] \\ u(x,0) = u(x,2\pi) \end{cases}$$

where $g \in C^1(Q \times R^1, R^1)$, with further additional assumptions.

Two kinds of results are obtained:

(1) The existence of at least three distinct solutions of the equation (0.1). The proof does not depend on any kind of symmetry assumptions on g .

(2) In case g is odd, an estimate of the number of solutions of (0.1), according to the asymptotic behavior of g at $u = 0$ and at $u = \infty$.

In case (1), the topological lemma we need is due to A. Castro, A. C. Lazer [4] and K. C. Chang [5], which reads as follows.

Lemma. *Suppose that f is a C^2 -real-valued function defined on R^n . Assume that f is bounded below, satisfies the Palais-Smale condition; and that θ is a nondegenerate, nonminimum critical point of f ; then there exist at least three distinct critical points of f .*

A typical result in this case is the

Theorem 1. *Suppose that $g \in C^1(Q \times R^1, R^1)$ satisfies the following conditions:*

- (g₁) $\exists \beta, 0 < g'_\xi(x,t,\xi) < \beta, g(x,t,0) = 0, g(x,t,\pm\infty) = \pm\infty$
- (g₂) $\exists \rho, g(x,t,\xi)/\xi > \rho > 3$ when $|\xi|$ is small enough uniformly in $(x,t) \in Q$.
- (g₃) $\exists \gamma, g(x,t,\xi)/\xi < \gamma < 3$ when $|\xi|$ is large enough uniformly in $(x,t) \in Q$.
- (g₄) $g'_\xi(x,t,0)$ is independent of $(x,t) \in Q$, and it does not belong to the spectrum of $-\square$; i.e. $g'_\xi(x,t,0) \notin \{k^2 - j^2: (k,j) \in Z \times N^*\}$.

Then the equation (0.1) has at least three distinct solutions.

In case (2), according to Clark [6], the two numbers (if they exist)

$$i_1(f) = \lim_{a \rightarrow -0} \gamma(f_a)$$