

# *Trace Ideal Criteria for Hankel Operators and Commutators*

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## **I. Introduction and Summary**

We present conditions for Hankel operators and related operators defined on the Hardy space  $H^2(\mathbf{R})$  to be in the Schatten ideals  $\mathcal{S}_p$ ,  $p \geq 1$ . The criterion is that the symbol have derivative in an appropriate Bergman space (i.e., that the symbol be in a certain Besov space). Our results are analogous to and offer a new approach to recent results of V. V. Peller involving Hankel operators defined on the Hardy space of the circle.

Background, definitions, and the statement of the main theorem are in Section 2. The proof of the theorem is in Section 3. It is based on Fourier transform calculations and the molecular decomposition of functions in Bergman spaces. Variations on the main theorem are given in Section 4. These include trace ideal criteria for various types of commutators and analogous results for operators defined using weighted projections. We also obtain conditions under which weighted projections are trace ideal perturbations of ordinary projections. In the final section, we mention a few open questions.

## **II. Preliminaries**

**A. Function spaces.** We denote by  $H^2$  the Hardy space consisting of those functions  $F$  which are holomorphic on the upper half plane  $U$  and for which

$$\|F\|_2^2 = \sup_{y>0} \int_{-\infty}^{\infty} |F(x + iy)|^2 dx$$

is finite. We will identify such an  $F$  with its boundary values. The Fourier transform,  $\hat{F}$ , of such a function is square integrable on the line and vanishes on the negative half line. Thus we will regard  $\hat{F}$  as an element of  $L^2(0, \infty)$ . The Fourier transform is a unitary map of  $H^2$  onto  $L^2(0, \infty)$ . We denote the inverse Fourier transform of  $g$  by  $\check{g}$ . For further discussion of these issues, see [16].

For  $p, r$  with  $0 < p < \infty$ ,  $-1/2 < r$ , we define the Bergman space  $A^{p,r}$  to be the space of functions  $F$  holomorphic in  $U$  for which