

Global Behavior of Solutions of Nonlinear Equations Depending on Infinite-Dimensional Parameters

J. C. ALEXANDER & STUART S. ANTMAN

1. Introduction. Let A and X be real Banach spaces, let D be a subset of $A \times X$, and let $f: D \rightarrow X$ be continuous. Let $\|\cdot\|$ denote the norms of both A and X ; the context will make it clear which is meant. We study the family of operator equations

$$(1) \quad x = f(a, x)$$

parametrized by an element a of the *parameter space* A . Equations like (1.1) arise throughout mathematics and the mathematical sciences. In virtually every previous global analysis of equations of the form (1.1) A is taken to be finite-dimensional. In fact, in very few of such analyses does the dimension of A exceed 1. Here we study (1.1) when A is infinite-dimensional. A pair (a, x) in D that satisfies (1.1) is called a *solution pair* of (1.1). We seek to describe the global topological properties of connected families of solution pairs of (1.1). After we obtain general theorems for important classes of equations of the form (1.1) we apply them to concrete examples of some physical importance.

Let us observe that in place of (1.1) we could study the ostensibly simpler problem $x = f(x)$, but now identify the operator f itself as the parameter. This approach is equally valid from the viewpoint of applications; the constructions underlying our basic proofs are simpler, however, in the context of (1.1). Moreover, the use of (1.1) allows us to effectively delimit the class of operators to be studied in a given problem.

Let us briefly recall some of the central results for the most frequently studied case of (1.1), namely that for which $A = \mathbf{R}$. Let $(b, y) \in D$ and let f be continuously differentiable near (b, y) . Let I be the identity map on X and let f_x denote the partial (Fréchet) derivative of f with respect to its second argument. If $I - f_x(b, y)$ is nonsingular, then the Implicit Function Theorem (based on the Contraction Mapping Principle) ensures that near (b, y) the solution pairs of (1.1) lie on a curve $\{(a, x): x = \hat{x}(a)\}$ with \hat{x} continuously differentiable and with $y = \hat{x}(b)$. If, furthermore, $f(a, \cdot)$ is compact for all a , then the global behavior of the connected set of solution pairs containing this curve can often be determined by