

Continuity of Weak Solutions to a General Porous Medium Equation

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1. Introduction

The aim of this paper is to extend the results I obtained in a previous work [10] about the continuity of weak solutions of singular parabolic equations in divergence form of the type

$$(1.1) \quad \frac{\partial}{\partial t} \beta(u) - \operatorname{div} \vec{a}(x, t, u, \nabla_x u) + b(x, t, u, \nabla_x u) \ni 0$$

in the sense of distributions over a domain Q in \mathbf{R}^{N+1} , $N \geq 1$. In [10] I considered the case of β being a coercive, maximal monotone graph in $\mathbf{R} \times \mathbf{R}$ with a jump at the origin, namely

$$(1.2) \quad \beta(s) = \begin{cases} \beta_1(s), & s > 0 \\ [-\nu, 0], & s = 0 \\ \beta_2(s) - \nu, & s < 0, \end{cases}$$

where β_i , $i = 1, 2$ are increasing coercive Lipschitzian functions in \mathbf{R} , and ν is a given positive constant. The situation was typical of diffusion processes with a change of phase. Here we consider the case of β continuous, coercive, monotone in \mathbf{R} , such that $\beta'(s)$ “blows-up” at $s = 0$. The model example of β I have in mind is

$$(1.3) \quad \beta(s) = |s|^{1/m} \operatorname{sign} s, \quad m > 1,$$

which occurs in filtration of gases in porous media when the flow obeys a polytropic regime.

Our goal is to prove that weak solutions (in a sense to be made precise) of (1.1), for β such as (1.3), are continuous.

Related regularity results with available knowledge essentially deal with non-negative weak solutions of

$$(1.4) \quad \beta(u)_t - \Delta u = 0 \quad \text{in } \mathcal{D}'(\Omega_T),$$

where Ω_T is a cylindrical domain in \mathbf{R}^{N+1} .

For $N = 1$, $\Omega_T \equiv \mathbf{R} \times \mathbf{R}^+$, the sharpest results are due to D. Aronson [1], [2],