

Toral Actions on 5-manifolds (II)

HAE SOO OH

Section 0. Introduction. A. Markov (1960) proved that a general classification of 5-manifolds is impossible. It does seem reasonable however to hope for results in case of 5-manifolds under some restrictions. S. Smale (1962) classified closed simply connected smooth 5-manifolds with vanishing second Stiefel-Whitney class. D. Barden (1965) had a complete classification of closed simply connected 5-manifolds.

In a series of papers, we try to classify closed orientable smooth 5-manifolds M supporting smooth effective actions of the 3-dimensional torus group, T^3 . By applying a technique similar to Orlik-Raymond [9], M can be broken down into elementary building blocks, $A(p, q, r)$, $B(a, b, c; p, q, r)$, C , and the T^3 -manifolds D having a 2-disk as an orbit space. The manifolds D were identified in [7], the first part of the investigation of M . We identify the building blocks, $A(p, q, r)$ and $B(a, b, c; p, q, r)$ in Sections 1 and 2 respectively. The building block C will be discussed in a third paper in preparation. Unlike the situation dealt with by Orlik-Raymond [9], here there are some difficulties in assembling these building blocks as a connected sum. But in some nice cases we can express M as a connected sum of these building blocks and some well-known manifolds. We discuss this in Section 3.

For definitions and terminologies, we refer to [7]. Here we restate the notations which are frequently used in this article: For relatively prime integers p, q, r , $G(p, q, r)$ means a circle subgroup of T^3 defined by $G(p, q, r) = \{(pt, qt, rt) : 0 \leq t < 1\} \pmod{Z^3}$. The S^n -bundles over S^2 are classified by $\pi_1(SO(n+1)) = Z_2$, if $n > 2$. The non-trivial bundle is denoted by $S^2 \widetilde{\times} S^n$. Suppose M_1 and M_2 are T^3 -manifolds of dimension 5. By $M_1 = M_2$, we mean that there exists a weakly equivariant diffeomorphism $f: M_1 \rightarrow M_2$. (That is, there is $\alpha \in \text{Aut}(T^3)$ such that $f(g \cdot m) = \alpha(g) \cdot f(m)$, $g \in T^3$, $m \in M$.) We write $M_1 \approx M_2$ when they are only topologically equivalent. The connected sum $M_1 \# M_2$ is obtained by removing the interior of 5-ball from each and attaching the resulting manifolds together along boundaries.

The interior of a tubular neighborhood $S^1 \times D^4$ of an embedded circle S^1 can be removed from M_1 and M_2 . The resulting manifolds can be attached along $S^1 \times S^3$ to obtain a new manifold $M_1 \#_S M_2$. This construction on manifolds was used by Goldstein-Lininger [3]. They showed that if M_1 and M_2 are simply connected then $M_1 \#_S M_2$ is either $M_1 \# M_2 \# (S^2 \times S^3)$ or $M_1 \# M_2 \# (S^2 \widetilde{\times} S^3)$. But