

Orbits Near Triple Collision in the Three-Body Problem

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Introduction. Analytical methods developed by Sundman, Siegel, and McGehee make it possible to study the qualitative behavior of orbits of the three-body problem as they pass through a neighborhood of the triple collision singularity.

In the case of non-zero angular momentum, triple collision is impossible and orbits passing sufficiently near the singularity exhibit the following behavior: one side of the triangle formed by the three bodies remains bounded and shorter than the other two throughout the motion while the longer sides become unbounded in both forward and backward time. Section 2 contains an account of these results, which are due to Sundman and Birkhoff.

The case of zero angular momentum is much different. Sundman showed that orbits which experience triple collision tend to one of five asymptotic configurations, the well-known central configurations. Siegel proved that the set of orbits tending to a given central configuration is an analytic manifold. One of our main results (Proposition 5.1) tells how these five manifolds fit together. In contrast to the case of non-zero angular momentum, orbits which do not actually collide can exhibit startling changes of configuration as they pass through a neighborhood of the singularity (see Figure 4).

Sections 1 and 3 present the technical results underlying the qualitative work, including the construction of a “collision manifold” which reflects the behavior of orbits near collision. This approach, which is due to McGehee, leads quickly to the above-mentioned results of Sundman and Siegel. Section 4 is devoted to a more detailed discussion of the flow on the collision manifold. The implications for orbits close to collision are presented in Section 5.

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1. Three-body problem in \mathbf{R}^3 . Let P_k , $k = 1, 2, 3$, be point particles in \mathbf{R}^3 with positions q_k , momenta p_k and masses m_k . Let q, p be the nine-dimensional vectors $(q_1, q_2, q_3), (p_1, p_2, p_3)$.

The three-body problem in \mathbf{R}^3 is governed by the Hamiltonian function

$$H(p, q) = \frac{1}{2} p \cdot M^{-1} p - U(q)$$